



Theoretical Distribution Fitting Of Monthly Inflation Rate In Nigeria From 1997- 2014

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ABSTRACT

In Nigeria, inflation has been a major economic issue with enormous detrimental effect on all the sectors of the economy. It affects economic growth and development. This study examined the probability distribution that best described the all items (12 month average change) inflation rate in Nigeria between 1997- 2014. The study made use secondary data collated from Central Bank of Nigeria (CBN) Statistical Bulletin 2015 on all items (12 month average change) of inflation rate in Nigeria. Four theoretical statistical distributions were fitted via Normal distribution, Logistic distribution, Log – Logistic distribution and 3-parameter Log-Logistic distribution. The 3-parameter Log-Logistic fitted the data as confirmed by Kolmogorov Simonov goodness of fit test and the probabilities of occurrence of inflation was obtained from the best fit distribution. The analysis showed that probability that inflation rate in Nigeria will lie between 0 and 20 is 0.954, which is very high and this implies that the probability of obtaining high inflation rate in the nearest future is very high.

Keywords: Inflation, Probability, Statistical Distribution, goodness-of-fit test

1.0 INTRODUCTION

Globally, Inflation has remained a major economic issue because of its effect in all the sectors of the economy, especially in developing countries. Bronfenbrenner and Holzman (1963) viewed inflation as a condition of generalized excess demand, in which too much money chases too few goods. In this definition, the emphasis is on the causes of inflation. Bronfenbrenner and Holzman (1963) further defined inflation as fall in the external value of money measured by foreign exchange rates, by the price of gold, or indicated by excess demand for gold or foreign exchange at official rates. This definition emphasizes external development (Frisch, 1983). Laidler and Parkin (1975) defined it as a process of continuously rising prices, or equivalently, of continuously falling value of money. There are several types of inflation, they are moderate inflation, creeping inflation, galloping inflation, hyper-inflation, open inflation and suppressed inflation. Inflation is caused by so many factors. Therefore, causes of inflation can be instigated from the demand side or the supply side factors. The demand side factors are caused by excess of aggregate demand for goods and services over supplies at a given price in time. There are many reasons that could lead to excess demand for goods and services. At any rate, the supply side factors are caused by that tend to reduce the supply of goods and services which are likely to affect the general price level in an economy.

Inflation in a developing country like Nigeria affects almost every facet of their economy and its attendant's costs are high. The degree of effects of inflation depends if it was an anticipated or unanticipated inflation. A correctly anticipated inflation is when the households and business firms can accurately predict the future inflation rate at all times. However, if they cannot predict the future inflation rate it becomes unanticipated inflation. Thus, in reality, it is a herculean task to accurately predict any kind of inflation correctly.

That notwithstanding, there are several measures to control inflation based on economic theory. These measures can be divided into monetary policy and fiscal policy measures.

Under monetary policy actions, adoption of suitable policy regarding interest rate and availability of credit is the emphasis. The major objective of these measures is to reduce the aggregate spending. Some of the measures are, first, credit control, the monetary authority of a country can increase the bank rates, sell securities in the open market, and raise the cash reserve ratio of banks. Second, demonetization of the currency is another policy measure to demonetize currency of higher denominations especially when there is a lot of black money in the system. Finally, issue of new currency can be used by the monetary authorities, though it is an extreme measure. Under this, a new note is exchanged for a number of notes of the old currency. This is usually adopted during hyper-inflation periods (Jhingan, 2010).

The fiscal measures are used to complement the monetary policy processes to control inflation. Some the fiscal measures include, first, is to increase taxes, this is to cut personal consumption expenditure, but the tax should not be so high to discourage savings and also, import duties can be reduced, while export duties increased. These measures are unavoidable to be effective in controlling inflation. Another fiscal measure is for government to apply surplus budget mechanism. This is regarded as an anti-inflationary tool; therefore government should drastically reduce deficit budget financing and adopt a surplus budget which encourages collecting more in revenues and spending less.

Nevertheless, other measures that could be adopted for inflation control include increasing production of consumer goods. Freezing of wages, when there is wage-price spiral in the system and more so, price control can be adopted as a direct measure to check inflation (Jhingan, 2010).

In Nigeria, many research works have been done in forecasting monthly rate using various methods such as time series analysis, regression analysis, econometric models and many others but none of these methods used have been able to fully model the monthly inflation rate. Hence, in this research work, parametric distribution modeling was used to model monthly inflation rate in Nigeria. This research work covers only the theoretical distribution fitting of monthly inflation rate in Nigeria. This study used the parametric distribution modeling to model inflation rates in Nigeria and four distributions (Normal, logistic, log- logistic, 3 parameter log – logistic) were fitted theoretically to the monthly inflation rate data spanning January 1997 –December 2014.

In this regard, the main objectives of this work are to use parametric distribution modeling to obtain the distribution that best describes inflation rate in Nigeria and also, to use the best fit statistical distribution to forecast the monthly inflation rate in Nigeria.

2.0 LITERATURE REVIEW

A lot of empirical research works have been done in the area of modeling and forecasting inflation using different methods

Otu et al. (2014), used the Box-Jenkins Seasonal Autoregressive Integrated Moving Average to analyze monthly inflation rates in Nigeria from October 2003 to November 2013. Forecast for the period of November, 2013 to November 2014 was made. ARIMA (1, 1, 1) (0, 0, 1)² was developed. The forecast results revealed a decreasing pattern of inflation rates in the first quarter of 2014 and turning point at the beginning of the second quarter of 2014, where the rates takes an increasing trend till September.

It was found that inflation showed volatility starting from 2006. According to them, the volatility in Nigeria's inflation series can be attributed to several economic factors. Some of these factors are money supply, exchange rate depreciation, petroleum prices increases, and poor agricultural production.

Charemza et al. (2014), fitted weighted skew normal distribution to monthly and annual inflation of 38 countries. It was found that skew normal distribution fits inflation forecast errors better than the two – piece normal distribution which is often used for inflation forecasting.

Omekara et al (2013), explores the frequency domain approach to model inflation rates as a time series, using period gram and Fourier series analysis methods, because of their simple way of modeling seasonality and eliminating significant peaks without re – estimating the model. Based on the analysis, it was observed that there exist both short term and long term cycles with period under study (“All-items

inflation rates from 2003 – 2011). This goes a long way to buttress that inflation is influenced by cyclical or periodic variation.

Osarumwense and Waziri (2013), explores the univariate non- linear time series analysis to the inflation data spanning from January,1995 to December, 2011.(GARCH (1,0) + ARMA (1,0)) model was used and 24 months forecast from January,2012 to December 2014 was made. From the analysis, the descriptive statistics for the monthly CPI rate and return shows a standard deviation of 29.13 from the series is high with a general mean of 58.550. skewness of 0.792 implies that the distribution is positively skewed with long right tail and a deviation from Normality. A kurtosis of -0.541 suggest flatness of the distribution.

Etuk et al. (2012), used a multiplicative seasonal autoregressive integrated moving average (ARIMA) model, (0, 1, 1) x (0, 1, 1)₁₂ to forecast monthly inflation rate in Nigeria for the data for inflation rates – All items (Year on change) – from 2003 to 2011. It was observed that monthly inflation rate in Nigeria is seasonal and forecast for 2012 was made.

Mordi et al. (2007), in their study of the best models to use in forecasting inflation rates in Nigeria identified areas of future research on inflation dynamics to include re – identifying ARIMA models, specifying and estimating a P- star model, others that can be used to forecast inflation with minimum mean square error.

Blix and Sellin (2000), derived a bivariate distribution for inflation with a given marginal distributions for inflation and output growth, using the translation method. The marginal distributions for inflation and output growth are derived from uncertainty in the macro variables that are deemed to be important for future inflation and output growth. The bivariate distribution makes it possible to discuss inflation forecast conditional on specific assumptions regarding the growth rate of output. It was found that the inflation and output forecast are two – piece normally distributed.

Stockton and Glassman (1987), conducted a comparative study on three different inflation processes namely rational expectations model, monetarist model and the expectation augmented Philips curve that are based on economic theory relationships that explain and form inflation. These processes were compared to one another, utilizing their out-of-sample forecast performance on an eight-quarter horizon and in addition a simple Autoregressive Integrated Moving Average (ARIMA) model was used as a bench mark to substantiate the theoretical validity of the econometric models. Their findings showed that the ARIMA model outperformed both the rational expectations model and the monetarist model and was found to perform just as good as the Philips curve in all specifications. They concluded that despite all theoretical efforts to explain causes of inflation, a simple ARIMA model of inflation turned in such a respectable forecast performance relative to the theoretically based specifications.

3.0 METHODOLOGY

In modeling the monthly inflation rate in Nigeria, the distributions (Normal, Logistic, and Log – Logistic and 3 parameter Log – Logistic) were fitted theoretically using the easy fit software and the best fit was determined by the Kolmogorov – Simonov goodness of fit test. The best fit was used to make forecast. The method of maximum likelihood estimation was used to estimate the parameters of the statistical distributions. (I am not comfortable with your language style of writing your methodology)

3.1 Normal Distribution

Normal distribution is a continuous probability distribution. The probability density function is symmetric about its mean and is non- zero over the entire real line and this distribution cannot be used to model right-skewed or left skewed data. It is not a good fit for bounded and non negative data. The normal distribution is belled shaped. It is applied in the natural and social sciences to represent real – valued random variable whose distributions are not known.

The probability function is given as;

$$f(x; \mu; \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$$

Where μ = mean or expectation

σ = standard deviation

x is a random variable

Properties of Normal Distribution

- Its shape is symmetric.
- Its distribution has a bump in the middle, with tails going down and out to the left and right.
- The mean, mode and the median are the same and lie directly in the middle of the distribution.
- Its standard deviation measures the distance on the distribution from mean to the inflection point.

Mean = μ is the value at the exact center of the normal distribution.

Mode = μ

Median = μ

Standard deviation = σ

Variance = σ^2

Skewness = 0

Ex. Kurtosis = 0

3.2 Logistic Distribution

The logistic distribution is a continuous probability distribution. It resembles the normal distribution in shape but has heavier tails (higher kurtosis)..

The probability density function is given as;

$$f(x; \mu; S) = \frac{e^{-\frac{x-\mu}{S}}}{S \left(1 + e^{-\frac{x-\mu}{S}}\right)^2} = \frac{1}{4S} \operatorname{sech}^2 \left(\frac{x-\mu}{2S} \right) \quad -\infty < x < \infty, S > 0$$

Where x is the random variable

μ is the mean

S is a scale parameter proportional to the standard deviation.

Properties of Logistic Distribution

- The logistic distribution has no shape parameter. This means that the probability density function has only one shape, the bell shape, and this shape does not change. The shape of the logistic distribution is similar to the normal distribution.
- The mean, μ is also the location parameter of the logistic probability density function, as it locates the pdf along the abscissa. It can assume value of $-\infty < \mu < \infty$.
- As μ decreases, the pdf is shifted to the left.
- As μ increases, the pdf is shifted to the right.
- As σ decreases, the pdf pushed towards the mean, or it becomes narrower and taller.
- As σ increases, the pdf spreads out away from the mean, or it becomes broader and shallower
- The scale parameter can assume values of $0 < \sigma < \infty$

Mean = μ

Median = μ

Mode = μ

Variance = $\frac{S^2 \pi^2}{3}$

Skewness = 0

Ex. Kurtosis = 1.2

3.3 Log- Logistic Distribution

The log – logistic distribution is a continuous distribution for a non – negative random variable. It is the probability distribution of a random variable whose logarithm has a logistic function. It is similar in shape to the log – normal distribution but has heavier tails.

It has various applications, namely;

- It is used in survival analysis as a parametric model for events whose rate increases initially and decreases later.
- It has been used in hydrology to model stream flow and precipitation.
- Applied in economics as a simple model of the distribution of wealth or income.

The probability density function of log – logistic distribution is given as;

$$f(x; \alpha; \beta;) = \frac{\left(\frac{\beta}{\alpha}\right) \left(\frac{x}{\alpha}\right)^{\beta-1}}{\left(1 + \left(\frac{x}{\alpha}\right)^\beta\right)^2} \quad 0 < x < \infty, \alpha > 0, \beta > 0$$

Where x is a random variable

α = a scale parameter

β = a shape parameter

Properties of Log-Logistic Distribution

- The parameter $\alpha > 0$ is a scale parameter and is also the median of the distribution.
- The parameter $\beta > 0$ is a shape parameter.
- The distribution is unimodal when $\beta > 1$ and its dispersion decreases as β increases.
- The shape of the log logistic is very similar to that of the lognormal distribution and the weibull distribution

$$\text{Mean} = \frac{\alpha \left(\frac{\pi}{\beta}\right)}{\sin\left(\frac{\pi}{\beta}\right)} \quad \beta > 1$$

$$\text{Median} = \alpha$$

$$\text{Mode} = \alpha \left(\frac{\beta-1}{\beta+1}\right)^{1/\beta}$$

If $\beta > 1, 0$ otherwise

$$\text{Variance; Var}(x) = \alpha^2 \left(\frac{2b}{s2b} - \frac{b^2}{\sin^2 b} \right), \beta > 2$$

3.4 Three Parameter Log Logistic

The three parameter log logistic distribution is a probability distribution which can be obtained from the log logistic distribution by addition of a shift parameter ε . thus if x has a log logistic distribution then $x + \varepsilon$ has a shifted log logistic distribution. So y has a shifted log logistic distribution if $\log(y - \varepsilon)$ has a logistic distribution. The shift parameter adds a location parameter to the scale and shape parameter of the (unshifted) log logistic.

The distribution is used in hydrology for modeling flood frequency.

The properties of the three parameter log logistic are derived from the log logistic distribution,

The probability density function is given as;

$$f(x; \mu; \sigma; \varepsilon) = \frac{\left(1 + \frac{\varepsilon(x-\mu)}{\sigma}\right)^{-\left(\frac{1}{\varepsilon+1}\right)}}{\sigma \left[1 + \left(\frac{\varepsilon(x-\mu)}{\sigma}\right)^{-\frac{1}{\varepsilon}}\right]^2} \quad \mu \in (-\infty, +\infty), \sigma \in (0, +\infty), \varepsilon \in (-\infty, +\infty)$$

$$x \geq \mu - \sigma / \varepsilon \quad (\varepsilon > 0)$$

$$x \leq \mu - \sigma / \varepsilon \quad (\varepsilon < 0)$$

$$x \in (-\infty, +\infty) \quad (\varepsilon = 0)$$

Where x = a random variable

μ = Mean.

σ = Standard deviation.

ε = Shifted parameter.

Where $\mu \in \mathbb{R}$ is the location parameter, $\sigma > 0$ the scale parameter and $\varepsilon \in \mathbb{R}$ the shape parameter

Mean = $\mu + \frac{\sigma}{\varepsilon} (\alpha \operatorname{csc}(\alpha) - 1)$ where $\alpha = \pi \varepsilon$

Median = μ

Mode = $\mu + \frac{\sigma}{\varepsilon} \left[\frac{(1 - \varepsilon)^{\varepsilon - 1}}{(1 + \varepsilon)} \right]$

Variance = $\frac{\sigma^2}{\varepsilon^2} \left[2\alpha \operatorname{csc}(2\alpha) - (\alpha \operatorname{csc}(\alpha))^2 \right]$ where $\alpha = \pi \varepsilon$

3.5 Maximum Likelihood Estimation

Likelihood Function

Definition; let x_1, x_2, \dots, x_n be a random sample of size n from a population with density function $f(x, \theta)$. The likelihood function of the sample values x_1, x_2, \dots, x_n , usually denoted by $L = L(\theta)$ is their joint density function given by:

$$L = f(x_1, \theta) f(x_2, \theta) \dots f(x_n, \theta) = \prod_{i=1}^n f(x_i, \theta). \quad \dots (1)$$

L gives the relative likelihood that the random variables assume a particular set of values x_1, x_2, \dots, x_n , for a given sample x_1, x_2, \dots, x_n , L becomes a function of the variable θ , the parameter

The principle of maximum likelihood consists in finding an estimator for the unknown parameter $\theta = (\theta_1, \theta_2, \dots, \theta_k)$, say, which maximizes the likelihood function $L(\theta)$ for variations in parameter, i.e., we wish to find $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_k)$ so that

$$L(\hat{\theta}) > L(\theta) \quad \forall \theta \in \Theta, \text{ i.e., } L(\hat{\theta}) = \sup L(\theta) \quad \forall \theta \in \Theta.$$

Thus if there exists a function $\hat{\theta} = \hat{\theta}(x_1, x_2, \dots, x_n)$ of the sample values which maximizes L for variations in θ , then $\hat{\theta}$ is to be taken as an estimator of θ . $\hat{\theta}$ is usually called maximum likelihood estimator (M.L.E). Thus $\hat{\theta}$ is the solution, if any, of

$$\frac{\partial L}{\partial \theta} = 0 \text{ and } \frac{\partial^2 L}{\partial \theta^2} < 0 \quad \dots (2)$$

Since $L > 0$, and $\log L$ is a non-decreasing function of L ; L and $\log L$ attain their extreme values (maxima or minima) at the same value of $\hat{\theta}$. The first of the two equations in (2) can be rewritten as;

$$\frac{1}{L} \frac{\partial L}{\partial \theta} = 0 \Rightarrow \frac{\partial \log L}{\partial \theta} = 0, \quad \dots (2a)$$

a form which is much more convenient from practical point of view.

If θ is vector valued parameter, then $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_k)$, is given by the solution of simultaneous equations:

$$\frac{\partial}{\partial \theta_i} \log L = \frac{\partial \log L}{\partial \theta_i}(\theta_1, \theta_2, \dots, \theta_k) = 0; \quad i = 1, 2, \dots, k \quad \dots (2b)$$

The above equation (2a) and (2b) are usually referred to as the likelihood equation for estimating the parameters. (Gupta S.C and Kapoor V.K, 2002)

3.6 Kolmogorov – Smirnov Goodness Of Fit Test

The Kolmogorov – Simonov test is a non-parametric test that is used to decide if a sample comes from a population with specific distribution. The K-S test statistic itself does not depend on the underlying

cumulative distribution function being tested. It is only apply to continuous distribution and it tends to be more sensitive near the center of the distribution than at the tails.

Test statistic

$$D = \max_{1 \leq i \leq N} \left(F(Y_i) - \frac{i-1}{N}, \frac{1}{N} - F(Y_i) \right)$$

Where F is the theoretical cumulative distribution of the distribution being tested which must be a continuous distribution and it must be fully specified (i.e. the location, scale, and shape parameters cannot be estimated from the data).

Significance level is α

Decision rule: the hypothesis regarding the distributional form is rejected if the test statistic, D, is greater than the critical value obtained from a table.

4.0 Analysis

4.1 Descriptive Analysis of the Monthly Inflation Rate (1997 – 2014)

Table 4.0 Descriptive Statistics for the data

Source: Author’s Computation

The data employed in this study comprises of 216 monthly observations of inflation rate in Nigeria.

Mean	11.63009
Standard error	0.30077
Median	11.50000
Mode	8.00000
Standard deviation	4.42033
Sample variance	19.53932
Kurtosis	0.25320
Skewness	0.25644
Minimum	0.90000
Maximum	26.70000
Sum	2512.10000

The summary statistics for the data in table 4.0 shows that standard deviation of 4.42033 from the series is low with a general mean of 11.63009. Skewness and kurtosis represent the nature of departure from normality. Positive or negative skewness indicate asymmetry. Skewness of 0.2564 implies that the distribution is positively skewed with a short right tail. Kurtosis of 0.2532 suggests flatness of the distribution and the series has a minimum inflation rate value of 0.90000 and a maximum value of 26.70000 giving inflation range of 25.8000.

4.2 Theoretical Distribution Descriptive Statistics

Table 4.1: Theoretical Descriptive Statistics

	Log-Logistic (3P)	Logistic	Normal	Log-Logistic
Mean	11.646	11.63	11.63	12.270
Median				
Mode	11.145	11.63	11.63	8.7615
Standard Deviation	4.5661	4.4203	4.4203	8.2948
Variance	20.849	19.539	19.539	68.803
Skewness	0.49058	0	0	10.679
Kurtosis	1.8039	1.2	0	N/A
Maximum	+INF	+INF	+INF	+INF
Minimum	-33.723	-INF	-INF	0

Source: Author’s Computation

Table 4.21 shows the descriptive statistics for each distribution. For log – logistic (3p) distribution, the summary statistics indicates a standard deviation of 4.5661 from the series with mean of 11.646. Skewness of 0.49058 indicates that the distribution is positively skewed to the right and kurtosis of 1.8039 suggests flatness of the distribution and a deviation from normality as shown in fig. 4.0. The summary statistics for the data and the distribution is approximately the same.

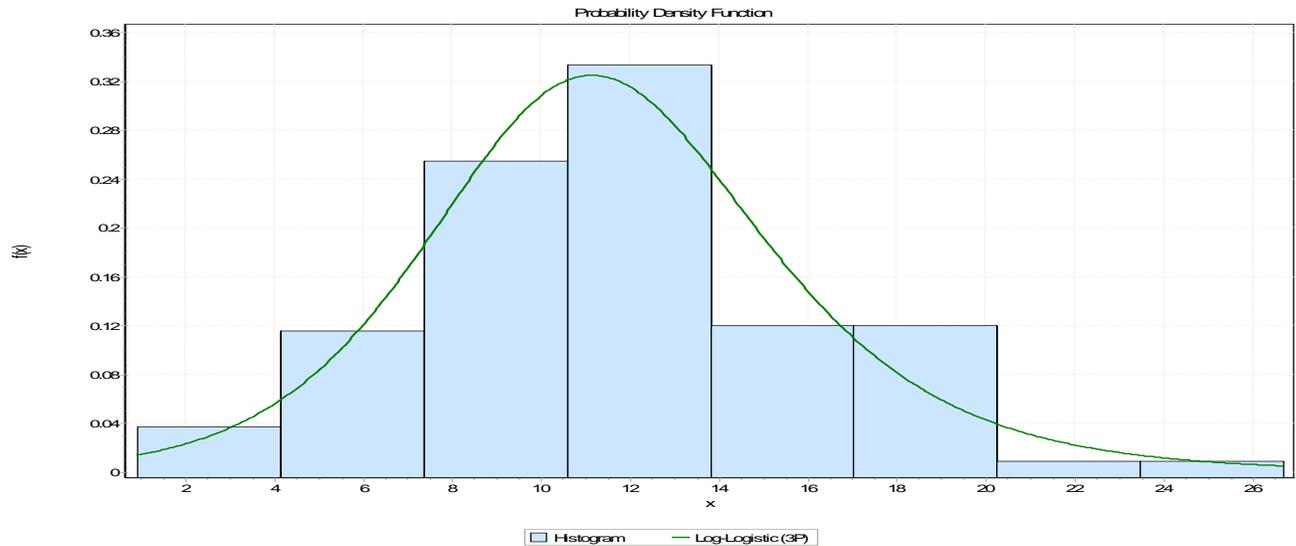


Fig. 4.0 Probability Density Graph of 3-Parameter Log-Logistic Distribution

Source: Author’s Computation

For logistic distribution, the summary statistics in table 4.1 shows a standard deviation of 4.4203 with mean of 11.63. It equally showed that the skewness is 0. The probability density graph of logistic distribution in fig. 4.1 shows that it fits the data. The kurtosis of 1.2 suggests flatness of the distribution as shown in fig 4.1.

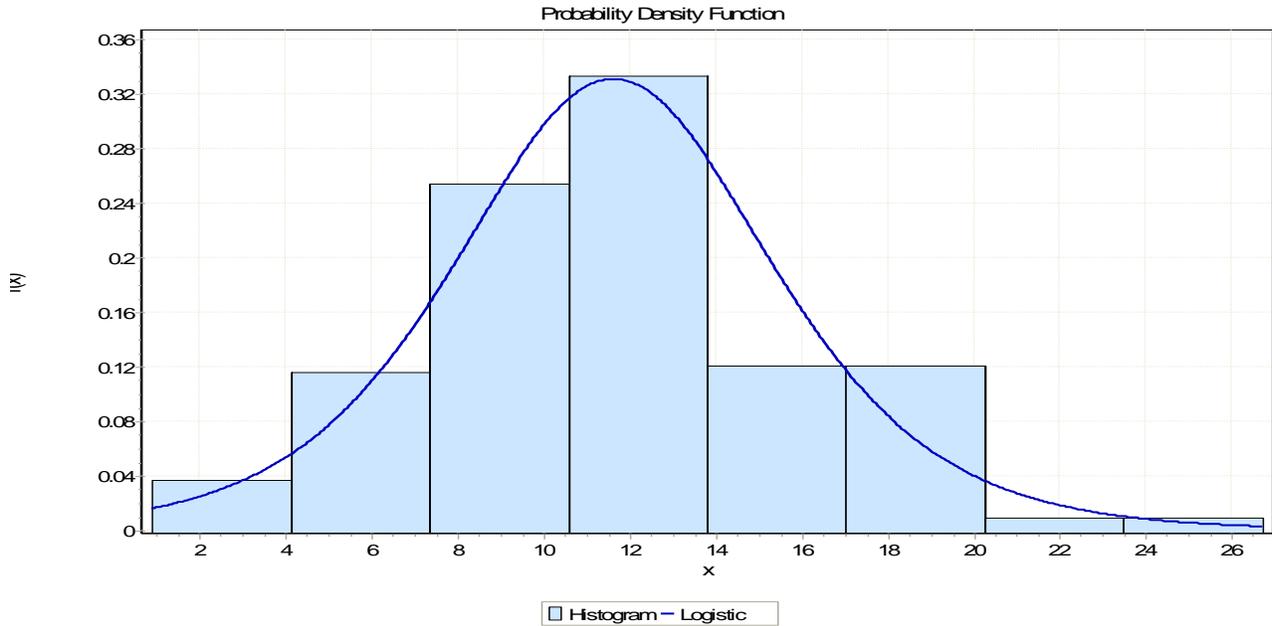


Fig. 4.1 Probability Density Graph of Logistic Distribution

Source: Author's Computation

In the case of normal distribution, the summary statistics shows a standard deviation of 4.4203 from the series is low with mean of 11.63. it also shows that both skewness and kurtosis are 0. The probability density graph in fig 4.2 indicates that the normal distribution fits the data.

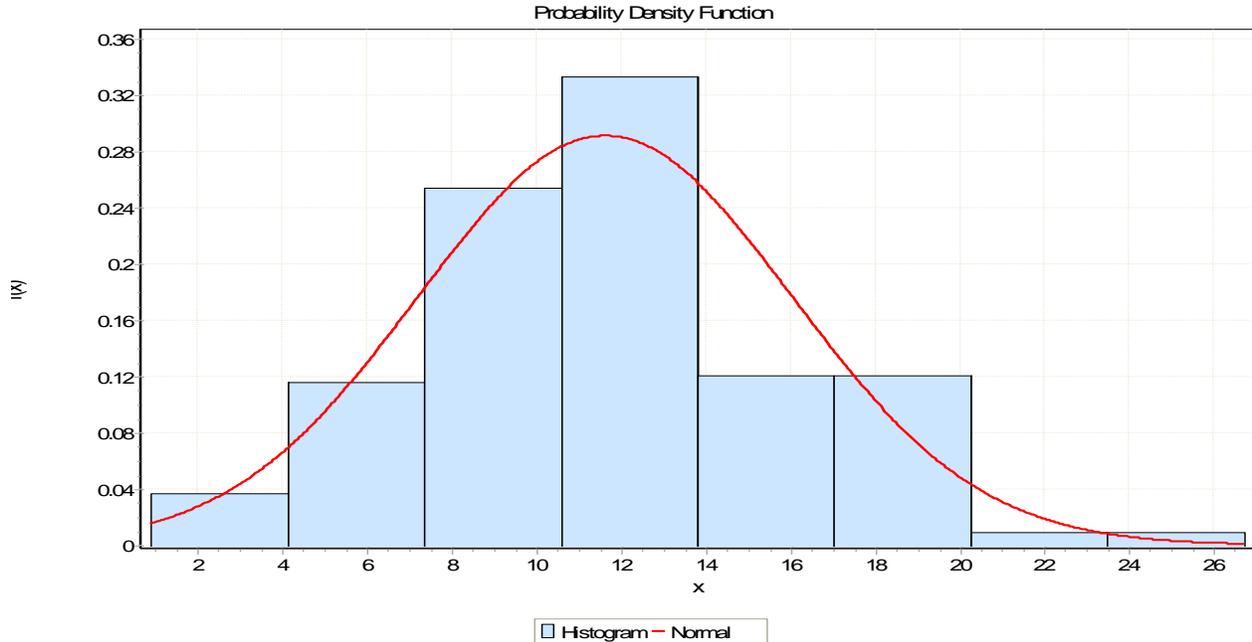


Fig. 4.2 Probability Density Graph of Normal Distribution

Source: Author's Computation

Also, for log – logistic distribution, the summary statistics shows a standard deviation of 8.2948 from the series is high with the mean of 12.270. Skewness of 10.679 and kurtosis of N/A. The probability density graph in fig. 4.3 indicates that log – logistic distribution does not fit the data as compared with the summary statistics for the data.

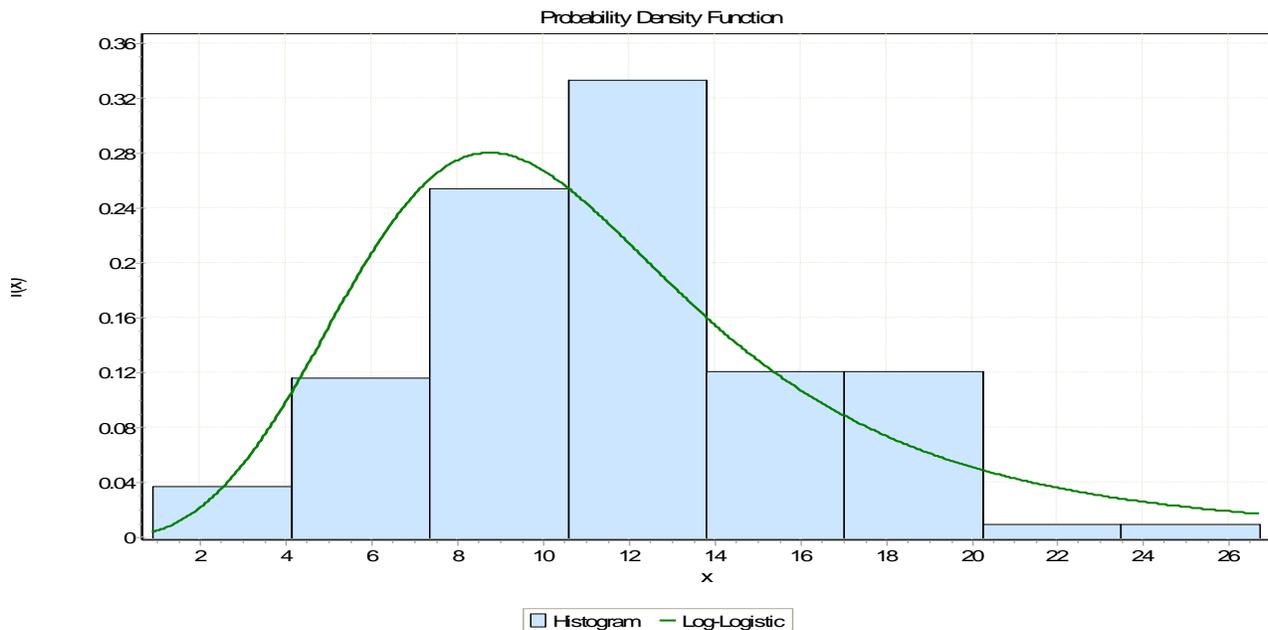


Fig. 4.3 Probability Density Graph of Log-Logistic Distribution

Source: Author's Computation

4.3 Goodness of Fit Test

Table 4.2: Goodness of Fit – Summary

#	Distribution	Kolmogorov Smirnov	
		Statistic	Rank
1	Log-Logistic	0.11359	4
2	Log Logistic (3P)	0.04841	1
3	Logistic	0.06097	2
4	Normal	0.06908	3

Source: Author's Computation

Table 4.2 shows the summary of the goodness of fit statistics for all the distributions fitted at 5% significance level. The rank column shows the ranking of the distributions according to the order of best fit. The distribution ranked 1 is the best followed by the one ranked 2, and it continues in that order. The Kolmogorov-Smirnov test ranked the distributions according to their estimations. However, we conclude that inflation rate in Nigeria follows a 3-Parameter Log-Logistic Distribution as confirmed by Kolmogorov Smirnov test.

4.4 Computed Probabilities from the Best Fit Distribution

In order to forecast the behavior of monthly inflation rate in Nigeria, the best fit probability distribution (3-parameter log-logistic) was used to compute the probabilities of inflation rate being within a certain bound, therefore:

$P(X < 10) = 0.35914$: This implies that the probability of obtaining a value of monthly inflation rate which is less than 10% in Nigeria is 0.3591 or 36% is very low.

$P(X < 5) = 0.05836$: This shows that the probability of obtaining a value of monthly inflation rate which is less than 5% in Nigeria is 0.05836 or 6% is very highly low.

$P(X > 10) = 0.64086$: This implies that the probability of obtaining a value of monthly inflation rate which is greater than 10% in Nigeria is 0.64086 or 64%, it is high.

$P(0 < X < 20) = 0.95409$: This implies that the probability of obtaining a value of monthly inflation rate which lies between 0% and 20% in Nigeria is 0.95409 or 95%, which is very high.

5.0 CONCLUSION

(A) This study conclude that high rate of inflation brings about economic instability which hinders economic growth and development.

(B) Based on the outcome of this study, it could be seen that the probability of obtaining a very high monthly inflation rate in time to come in Nigeria is very high.

(C) The 3-Parameter Log Logistic distribution fits the monthly inflation rate data, and the probability of an increase in monthly inflation rate in Nigeria is very high by using this distribution to model monthly inflation rate.

(D) Therefore, government should adopt appropriate monetary and fiscal policies mix to stem this ugly possibility

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