International Journal of Innovative Scientific & Engineering Technologies Research 6(4):10-22, Oct.-.Dec., 2018

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Construction of Balanced Incomplete Block Design of Lattice Series I and II

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ABSTRACT
There are different methods of constructing balanced incomplete block designs. The method used in this paper is the construction of lattice or orthogonal designs of series I and II due to Yates Algorithm and Khare and Federer. Yates Algorithms is the oldest method of constructing balance incomplete block design with parameters and $t = K^2 + K + 1, k = K, r = K, \lambda = 1$ for series I and II respectively with K is prime power. This design is only available when the number of treatment is a perfect square and the block size is the square root of this number. Khare and Federer is an alternative method of constructing lattice designs of series I and II. This design is available for both prime and non-prime power of K. By augmenting series I to Series II with parameter $K + 1$. Different examples were used for the construction and each of these example construct well with balance incomplete block designs of lattice series I and II.
Keywords: BIB Design,Yates Algorithms and Khare and Federer, Squared Design, Lattice series I and II,

1.0 INTRODUCTION

When the number of treatments is very large and blocking is must, the Incomplete Block Designs are generally used. The origins of incomplete block designs go back to Yates (1936) who introduced the concept of balanced incomplete block designs and their analysis utilizing both intra- and inter-block information Yates (1940). Other incomplete block designs were also proposed by Yates (1936, 1940), who referred to these designs as quasi-factorial or lattice designs. Further contributions in the early history of incomplete block designs were made by Bose (1939, 1942a, and 1942b) and Fisher (1940) concerning the structure and construction of balanced incomplete block designs. In order to eliminate heterogeneity; a concept of Balanced Incomplete Block (BIB) Design was introduced, which reduce heterogeneity to a greater extent than is possible with randomized block design and Latin square design. The importance of BIB designs in statistical design of experiments for varietal trials was, however, realized only in 1936 when Yates (1936) discussed these designs in the context of biological experiments. F. Yates (1936) introduced these designs in his paper, a new method of arranging variety trials, involving a large number of varieties.

Sharma and Kumar (2014) developed balanced incomplete block design using Hadamardrhotrices. Bayrak and Bulut (2006) constructed orthogonal balanced incomplete block design. Arunachalamet al. (2016) constructed of efficiency-balanced design. Pachamuthu (2011) showed construction of mutually orthogonal Latin square and check parameter relationship of balanced incomplete block design.

More recently different methods of constructing variance balanced and efficiency balanced block designs with repeated blocks have been given, such, Ghosh and Shrivastava (2001), Ceranka and Graczyk (2001, 2007, 2008 and 2009). Ghosh and Shrivastava (2001) developed the methods of construction of BIB designs with repeated blocks so as to distinguish the usual BIBD with repeated blocks.

Ceranka and Graczyk (2007) developed same new construction methods of the variance balanced block designs with repeated blocks. However from the practical point of view it may not possible to construct the design with equalize blocks accommodating the equireplication of each treatment in all the blocks.

Ceranka and Graczyk (2008) developed some new construction methods of the variance balanced block designs with repeated blocks, which are based on the specialized product of incidence matrices of the balanced incomplete block designs. From a practical point of view, it may not be possible to construct a design with equiblock sizes accommodating the equireplication of each treatment in all the blocks. In this paper researcher consider a class of block designs called variance balanced block designs which can be made available in unequal block sizes and for equal replications. Also Ceranka and Graczyk (2009) some new construction schemes of Efficiency Balanced block designs with repeated blocks for v treatments and some ways of admitting given design structures to construct new designs for other number of treatments.

We always need to set up a design in such a way that the variability in response due to uncontrolled variables (sometimes called experimental error) is not so great that it makes the effects of the controlled variables. We also want designs which are efficient, that is, designs where we can answer the questions of interest with a minimal amount of data because of the expense associated with data collection.

PROPERTIES OF BIB DESIGNS

The following relations hold among the parameters, and even these are only necessary conditions for the existence of a BIB design:

$$rt = bk \quad \ldots(1)$$

$$\lambda(t-1) = r(k-1) \quad \ldots(2)$$

$$r > \lambda \quad \ldots(3)$$

$$b \geq t \quad \ldots(4)$$

(2) An incomplete block design (IBD) with the same number of treatments and blocks is called a symmetrical incomplete block design. If t is even then $r-\lambda$ must be a perfect square for a symmetrical BIB design to exist. This is only one of several necessary conditions for the existence of a symmetrical BIB design.

Construction of Balanced Incomplete Block Designs

There is no single method of constructing all BIB designs. The research paper is limited to as quasi-factorial or lattice designs, since their construction is based on concepts of factorial experiments. Several important series of balanced incomplete block designs can be derived from orthogonal factorial designs. One is the series on *balanced lattice* of Yates (1936).

Lattice Designs

Lattice designs are an important class of incomplete block designs. These designs were originally introduced by Yates (1936). Here, we shall deal with square lattices.

Square Lattice Designs

The characteristics feature of these designs is that the number of treatments is a perfect square and the block size is the square root of this number. Moreover, incomplete blocks are combined in groups to form separate replications of the treatments are flexible in these designs and are useful for situations in which a large number of treatments are to be tested. If the design has two replications of the treatment, it is called a simple lattice; if it has 3 replications it is called a triple-lattice and so on. In general, if the number of replications is m, it is called an m-ple lattice. Square lattice designs can be constructed as follows:

ORTHOGONAL SERIES

Two Latin squares of the same size are orthogonal, if it well superimposed on each other every ordered numbers occur exactly once. This method is actually the oldest method. **These designs are due to Yates (1936b)** and are referred to as quasi-factorial or lattice designs, since their construction is based on concepts of factorial experiments.The designs generated by these methods have parameters; $t = K^2, k = K, b = K(K+1), r = K+1, \lambda = 1$ with K is prime or prime power. **This series of designs is also called orthogonal series 1 (OS1).**

Procedure For Construction Orthogonal Design of $K \times K$, K is Prime

(1) Arrange the first square in order.
(2) Obtain the next square by multiplying the element of the first row of the square by 2 modulus k.
(3) Arrange the column in order.
(4) Obtain the third square by multiplying the element of the first row of the first square by 3 modulus k.
(5) Then, arrange the column in order.
(6) Continue until the (k-1) square is obtained by multiplying the element of the first row of the first square by k-1 modulus k.

Lattice Design Series I (Orthogonal Series I)
The designs have the parameters: $t = K^2, k = K, b = K(K+1), r = K+1, \lambda = 1$

Procedure to Form A Lattice Square Design Of Series I By Yates

1) Form a standard square of size k.
2) Form a block using each row of the square.
3) Form a block using each column of the square
4) For each of the (k-1) orthogonal squares super impose the standard square on it.
5) All treatments pair with the same letter in standard squares makes up a block.

Example 1: Construct optimal design for lattice square design of series I

$t = 4, \quad k = 2, \quad b = 2(2+1) = 6, \quad r = 2+1 = 3, \quad \lambda = 1$

Orthogonal square

A		**Standard Array**	**Super Impose**		
0	1	0	1	0^0	1^1
1	0	2	3	1^2	0^3

Block

b_1	b_2	b_3	b_4	b_5	b_6
0	2	0	1	0	1
1	3	2	3	3	2

Example 2: Construct optimal design for lattice square design of series I

$t = 9, \quad k = 3, \quad b = 3(3+1) = 12, \quad r = 3+1 = 4, \quad \lambda = 1$

Orthogonal square

A			Ax2 (mod 3)			**Standard Array**		
0^0,	1^1	2^2	0^0,	2^1	1^2	0	1	2
1^3	2^4	0^5	1^3	0^4	2^5	3	4	5
2^6	0^7	1^8	2^6	1^7	0^8	6	7	8

Blocks

Treatment

1	2	3	4	5	6	7	8	9	10	11	12
0	3	6	0	1	2	0	1	2	0	2	1
1	4	7	3	4	5	5	3	4	4	3	5
2	5	8	6	7	8	7	8	6	8	7	6

Example 3: Construct optimal design for lattice square design of series I

$t = 25, \quad k = 5, \quad b = 5(5+1) = 30, \quad r = 5 + 1 = 6, \quad \lambda = 1$

Construct optimal design for lattice square design of series I.

Orthogonal Square

A					A x 2(mod 5)					A x3(mod 5)					A x 4(mod 5)					standard array				
0	1	2	3	4	0	2	4	1	3	0	3	1	4	2	0	4	3	2	1	0	1	2	3	4
1	2	3	4	0	1	3	0	2	4	1	4	2	0	3	1	0	4	3	2	5	6	7	8	9
2	3	4	0	1	2	4	1	3	0	2	0	3	1	4	2	1	0	4	3	10	11	12	13	14
3	4	0	1	2	3	0	2	4	1	3	1	4	2	0	3	2	1	0	4	15	15	17	18	19
4	0	1	2	3	4	1	3	0	2	4	2	0	3	1	4	3	2	1	0	20	21	22	23	24

Super Impose

0^{0}	1^{1}	2^{2}	3^{3}	4^{4}	0^{0}	2^{1}	4^{2}	1^{3}	3^{4}	0^{0}	3^{1}	1^{2}	4^{3}	2^{4}	0^{0}	4^{1}	3^{2}	2^{3}	1^{4}
1^{5}	2^{6}	3^{7}	4^{8}	0^{9}	1^{5}	3^{6}	0^{7}	2^{8}	4^{9}	1^{5}	4^{6}	2^{7}	0^{8}	3^{9}	1^{5}	0^{6}	4^{7}	3^{8}	2^{9}
2^{10}	3^{11}	4^{12}	0^{13}	1^{14}	2^{10}	4^{11}	1^{12}	3^{13}	0^{14}	2^{10}	0^{11}	3^{12}	1^{13}	4^{14}	2^{10}	1^{11}	0^{12}	4^{13}	3^{14}
3^{15}	4^{16}	0^{17}	1^{18}	2^{19}	3^{15}	0^{16}	2^{17}	4^{18}	1^{19}	3^{15}	1^{16}	4^{17}	2^{18}	0^{19}	3^{15}	2^{16}	1^{17}	0^{18}	4^{19}
4^{20}	0^{21}	1^{22}	2^{23}	3^{24}	4^{20}	1^{21}	3^{22}	0^{23}	2^{24}	4^{20}	2^{21}	0^{22}	3^{23}	1^{24}	4^{20}	3^{21}	2^{22}	1^{23}	0^{24}

b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}	b_{13}	b_{14}	b_{15}	b_{16}	b_{17}	b_{18}	b_{19}
0	5	10	15	20	0	1	2	3	4	0	1	2	3	4	0	3	1	4
1	6	11	16	21	5	6	7	8	9	9	5	6	7	8	7	5	8	6
2	7	12	17	22	10	11	12	13	14	13	14	10	11	12	14	12	10	13
3	8	13	18	23	15	16	17	18	19	17	18	19	15	16	16	19	17	15
4	9	14	19	24	20	21	22	23	24	21	22	23	24	20	23	21	24	22

b_{20}	b_{21}	b_{22}	b_{23}	b_{24}	b_{25}	b_{26}	b_{27}	b_{28}	b_{29}	b_{30}
2	0	2	4	1	3	0	4	3	2	1
9	8	5	7	9	6	6	5	9	8	7
11	11	13	10	12	14	12	11	10	14	13
18	19	16	18	15	17	18	17	16	15	19
20	22	24	21	23	20	24	23	22	21	20

LATTICE SQUARE DESIGN SERIES II

Example 4:Construct optimal design for t =7, k = 3

Solution

Condition: $t=k^2-k+1=3^2-3+1=7$, b = 7, r = 3, $\lambda=1$

Construction method: Lattice Square Series II

Step 1: we firstly construct orthogonal Series I with parameter

t = 4, k = 2, b = 6, r = 3, $\lambda=1$

Orthogonal squareStandard ArraySuper impose

0	1	0	1	0^0	1^1
1	0	2	3	1^2	0^3
b_1	b_2	b_3	b_4	b_5	b_6
0	**2**	**0**	**1**	**0**	**1**
1	**3**	**2**	**3**	**3**	**2**

Step II: Construct Orthogonal Series II by Augmenting Orthogonal Series I, with treatment left out from Orthogonal Series I (i.e. 4, 5, and 6). Adding two of each of them to each two block and the remaining treatment to last block

Augmented Lattices series I to Lattice series II

b_1	b_2	b_3	b_4	b_5	b_6	b_7
0	2	0	1	0	1	4
1	3	2	3	3	2	5
4	4	5	5	6	6	6

Example 5: Construct optimal design for t = 13, k = 4

Solution

Test for condition: $t = k^2 - k+1 = 4^2 - 4+1 = 13$, b=13, r=4, $\lambda = 1$

Method of construction: Lattice square design series II

Step 1: we firstly construct lattice square design series I with parameters of t=9, k=3, r=4, b=12 $\lambda = 1$

Orthogonal series

A			A x 2(mod 3)			**Standard Array**		
0	1	2	0	2	1	0	1	2
1	2	0	1	1	2	3	4	5
2	0	1	2	0	0	6	7	8

Super impose

0^0	1^1	2^2
1^3	2^4	0^5
2^6	0^7	1^8

0^0	2^1	1^2
1^3	0^4	2^5
2^6	1^7	0^8

Construction of lattice series I

b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
0	2	6	0	1	2	0	1	2	0	2	1
1	4	7	3	4	5	5	3	4	4	3	5
2	5	8	6	7	8	7	8	6	8	7	6

Step II: augmented series I to lattice series II with treatment left out from series I (9, 10, 11, and 12) and adding the last block

Construction of lattice series II

b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
0	2	6	0	1	2	0	1	2	0	2	1
1	4	7	3	4	5	5	3	4	4	3	5
2	5	8	6	7	8	7	8	6	8	7	6

An Alternative Method Of Constructing These Designs Is Given by Khare And Federer (1981).
This algorithm can be described as follows:

1)Write the treatment numbers 1, 2, . . ., t consecutively in a square array of K rows and K columns to yield replicate 1 of the resolvable design, with rows constituting the blocks.

2. Transpose the rows and columns of replicate 1 to obtain replicate 2.

3. Take the main right diagonal of replicate 2 to form the first row of replicate 3, and write the remaining elements in each column of replicate 2 in a cyclic order in the same column for replicate 3.

4. Repeat step 3 for replicate 3 to generate replicate 4.

5. Continue this process on the just generated replicate until $K + 1$ replicates have been obtained.

Example 6: Construct optimal design for lattice square design of series I

$t = 9, \quad k = 3, \quad b = 3(3+1) = 12, \quad r = 3 + 1 = 4, \quad \lambda = 1$

Rep I	**Rep II**	**Rep III**	**Rep IV**
1 2 3	1 4 7	1 5 9	1 6 8
4 5 6	2 5 8	2 6 7	2 4 9
7 8 9	3 6 9	3 4 8	3 5 7

Blocks

b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
1	4	7	1	2	3	1	2	3	1	2	3
2	5	8	4	5	6	5	6	4	6	4	5
3	6	9	7	8	9	9	7	8	8	9	7

Example 7: Construct optimal design for lattice square design of series I

$t = 16 = 4^2, \quad k = 4, \quad b = 20, \quad r = 5, \quad \lambda = 1$

Replicate I

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Replicate II

1	5	9	13
2	6	10	14
3	7	11	15
4	8	12	16

Replicate III

1	6	11	16
2	7	12	13
3	8	9	14
4	5	10	15

Rep IV

1	7	9	15
2	8	10	16
3	5	11	13
4	6	12	14

Rep V

1	8	11	14
2	5	12	15
3	6	9	16
4	7	10	13

Blocks

Treatment

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	5	9	13	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
2	6	10	14	5	6	7	8	6	7	8	5	7	8	5	6	8	5	6	7
3	7	11	15	9	10	11	12	11	12	9	10	9	10	11	12	11	12	9	10
4	8	12	16	13	14	15	16	16	13	14	15	15	16	13	14	14	15	16	13

Example 8: Construct optimal design for lattice square design of series I

$t = 25 = 5^2, \quad k = 5, \quad b = 30, \quad r = 6, \quad \lambda = 1$

Replicate I					**Replicate II**					**Replicate III**				
1	2	3	4	5	1	6	11	16	21	1	7	13	19	25
6	7	8	9	10	2	7	12	17	22	2	8	14	20	21
11	12	13	14	15	3	8	13	18	23	3	9	15	16	22
16	17	18	19	20	4	9	14	19	24	4	10	11	17	23
21	22	23	24	25	5	10	15	20	25	5	6	12	18	24

Replicate IV					**Replicate V**					**Replicate VI**				
1	8	15	17	24	1	9	12	20	23	1	10	14	18	22
2	9	11	18	25	2	10	13	16	24	2	6	15	19	23
3	10	12	19	21	3	6	14	17	25	3	7	11	20	24
4	6	13	20	22	4	7	15	18	21	4	8	12	16	25
5	7	14	16	23	5	8	11	19	22	5	9	13	17	21

Blocks

Treatment

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	6	11	16	21	1	2	3	4	5	1	2	3	4	5
2	7	12	17	22	6	7	8	9	10	7	8	9	10	6
3	8	13	18	23	11	12	13	14	15	13	14	15	11	12
4	9	14	19	24	16	17	18	19	20	19	20	16	17	18
5	10	15	20	25	21	22	23	24	25	25	21	22	23	24

Blocks

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
8	9	10	6	7	9	10	6	7	8	10	6	7	8	9
15	11	12	13	14	12	13	14	15	11	14	15	11	12	13
17	18	19	20	16	20	16	17	18	19	18	19	20	16	17
14	25	21	22	23	25	24	25	21	22	20	23	24	25	21

LATTICE DESIGN SERIES II BASED ON KHARE AND FEDERER
The designs have the parameters: $t = b = K^2 - K + 1, \quad k = K, \quad r = K, \quad \lambda = 1$

Example 9: Construct optional design for

$(i) t = 7, \quad k = 3 \quad (ii) t = 13, \quad k = 4 \quad (iii) t = 21, \quad k = 5$

Solution

$$(i) t = 7, \quad k = 3$$

We firstly construct Lattice series I

$t = 4, \quad k = 2, \quad b = 2(3) = 6 \quad r = 3, \quad \lambda = 1,$

Rep I		***Rep II***		***Rep III***	
1	2	1	3	1	4
3	4	2	4	2	3

Block

b_1	b_2	b_3	b_4	b_5	b_6	b_7
1	3	1	2	1	2	5
2	4	3	4	4	3	6
5	5	6	6	7	7	7

(ii)

$t = 13, \quad k = 4$

We firstly construct lattice Series I with parameters

$t = 9, \quad k = 3 \quad b = 12, \quad r = 4 \quad \lambda = 1,$

Rep I			**Rep II**			**Rep III**			**Rep IV**		
1	2	3	1	4	7	1	5	9	1	6	8
4	5	6	2	5	8	2	6	7	2	4	9
7	8	9	3	6	9	3	4	8	3	5	7

Block

b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}	b_{13}
1	4	7	1	2	3	1	2	3	1	2	3	10
2	5	8	4	5	6	5	5	4	6	4	5	11
3	6	7	7	8	9	9	7	8	8	9	7	12
10	10	10	11	11	11	12	12	12	13	13	13	13

Example 10: Construct optimal design for

$t = 21, \quad k = 5$

Solution

We firstly construct lattice at series I

$t = 16 = 4^2 \quad k = 4 \quad b = 20, \quad r = 5 \quad \lambda = 1,$

Replicate 1

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Replicate II

1	5	9	13
2	6	12	17
3	7	11	15
4	8	12	16

Replicate III

1	6	11	16
2	7	12	20
3	8	9	16
4	5	10	15

Replicate IV

1	7	9	15
2	8	10	8
3	5	11	12
4	6	15	16

Replicate V

1	8	11	14
2	5	12	15
3	6	9	16
4	7	10	13

Block

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	5	9	13	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	17
2	6	10	14	5	6	7	8	6	5	8	7	7	8	5	6	8	5	6	7	18
3	7	11	15	9	10	11	12	11	12	9	10	9	10	11	12	11	12	9	10	19
4	8	12	16	13	14	15	16	16	13	14	15	15	16	13	14	14	15	16	13	20
17	17	17	17	18	18	18	18	19	19	19	19	20	20	20	20	21	21	21	21	21

CONCLUSION

Construction of balanced incomplete block designs is achieved when every factor level combinations occur an equal number of times (r) in the design and each pair of factors occur together in an equal number (b) of block in the design. Different examples were used for the construction of lattice square of series I and II and each of the example construct well with balanced incomplete block designs.

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