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Modelling Volatility Transmission Between Crude Oil Price And The Nigerian Exchange Rate Using Multivariate GARCH Models

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ABSTRACT
This study examines the volatility transmission between crude oil price and the Nigerian exchange rate by employing multivariate GAR-CH modeling. This paper investigated the volatility transmission between the Nigerian exchange rate and crude oil price from January 2009 to December 2018 using three multivariate GARCH Models (BEKK, DCC, and CCC). All analyzed models show similar behavior in variances and covariances. In particular, the estimated conditional correlation with DCC model is negative and very weak, reflecting a weak negative relationship between the crude oil price and the Nigeria exchange rate. This shows that the oil price and exchange rate volatility tend to move in opposite directions. The result revealed that conditional covariances exhibit significant changes over time for both exchange rate and crude oil price. The performance evaluation carried out revealed that Bivariate GARCH-BEKK model effectively captures the volatility spillovers between crude oil price and exchange rate in Nigeria.
Keywords: Crude Oil Price, Exchange Rate, Volatility Spillovers, BEKK model, DCC Model, CCC model.

INTRODUCTION

Observation of Price processes at speculative markets such as foreign exchanges or stock, bond, or commodity markets have been attracting a huge interest among researchers for decades. A time series model that has been proven to approximate empirical price processes quite accurately is the random walk model (Fama 1965). According to this model the best forecast of a future price is today's price and, thus, the latter summarizes efficiently the available information for prediction. Although price processes are hardly predictable, the variance of the forecast error is time dependent and can be estimated by means of observed past variations. The development of econometrics has led to the invention of adaptive methods for modelling the mean value of the variable in question, the most widely used of which are the ARIMA methods (Box and Jenkins, 1970) and methods derived from them. The autoregressive-and moving average models were used to try to capture the volatility dynamics, and thereafter the Box-Jenkins-type ARMA-model gained attention as a method for capturing volatility movement. However, it was found by researchers and practitioners that the volatility suffered from ´volatility clustering´ and hence, other more sophisticated models were invented. The phenomenon of time-varying and future volatility is well known and has generated a wide expanse of econometric literatures such as Engle (1982), Bollerslev (1986), and Taylor (1986) who introduced the (generalized) auto regressive conditionally heteroskedastic GARCH process and the stochastic volatility model, respectively.
The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) method is one of the techniques based on the assumption that the random component of the model shows changes in

variability. It was developed in a simplified form by Engle (1982) and later generalised by Bollerslev (1986). The model was applied successfully in modelling the changing variability (or volatility) of the variable in time series, with the applications being taken in large measure from the area of financial investments. After identifying an asymmetric relationship between conditional volatility and conditional mean value, the econometrists focused their efforts on the design of methods for the modelling of this phenomenon. Nelson (1991) proposed an exponential GARCH (EGARCH) model, based on a logarithmic expression of the conditional variability in the variable under analysis. Later, a number of modifications were derived from this method. One of them is the TARCH method (Threshold ARCH), which was introduced by Zakoian (1994). Practical experience in this area was described by Bollerslev, Chou and Kroner in full detail (1992). The application of the GARCH model in the conditions of the Czech capital market was studied by Hančlová (2000). However, the ARCH-model had limitations due to its non-negatively constrains and the GARCH model was introduced by Bollerslev (1986) and Taylor (1986). According to Brooks, (2008), the GARCH-model is more parsimonious since it avoids over-fitting and is less likely to breach the non-negativity constraint than the ARCH-model. One of the limitations of univariate volatility models is that they model the conditional variance of each series independently of all other series and so in the case of "volatility spillovers" between markets or assets, the univariate model would be mis-specified. Also, the covariances between series are of mush interest, as well as the variances of each series. While univariate descriptions are useful and important, problems of risk assessment, asset allocation, hedging in futures markets and options pricing, portfolio Value at Risk estimates, CAPM betas, and so on require a multivariate framework. All the problems mentioned above require covariances as inputs. Multivariate GARCH models can potentially overcome both of these deficiencies of their univariate counterparts. In addition, there are many situations when empirical multivariate models of conditional heteroscedasticity can be used fruitfully (Brooks, 2002). One of the limitations of univariate volatility models is that they model the conditional variance of each series independently of all other series and so in the case of "volatility spillovers" between markets or assets, the univariate model would be mis-specified. Also, the covariances between series are of mush interest, as well as the variances of each series. While univariate descriptions are useful and important, problems of risk assessment, asset allocation, hedging in futures markets and options pricing, portfolio Value at Risk estimates, CAPM betas, and so on require a multivariate framework. All the problems mentioned above require covariances as inputs. Multivariate GARCH models can potentially overcome both of these deficiencies of their univariate counterparts. In addition, there are many situations when empirical multivariate models of conditional heteroscedasticity can be used fruitfully (Brooks, 2002).

This paper reconsiders the volatility transmission of exchange rate and crude oil price using multivariate GARCH models. By volatility transmission we mean how uncertainty in one market spreads to another. Often the current value of a variable depends not only on its past values, but also on past and/or current values of other variables (Schmidth, 2005). Exchange rate and oil price volatility embraces a numerous divergent of interest from academics, financial economists and decision makers and It is well known that price movements in crude oil can spread easily and instantly to the rate of exchange (Veysel and Cameer 2018). According to Liang and Minh (2012), high fluctuation on exchange rates and crude oil markets result in a more challenging trade execution, exposing traders in foreign markets to risks, huge capital requirement to invest into their business and decreasing the effectiveness of benchmark hedging relatively compare to other asset classes. This is so because exchange rates are seen determined by expected future fundamental conditions, among which crude oil is surely an important key player as supported by empirical evidence found in (see Yousefi and Wirjemto (2004), Krichene (2005), Zhang, fan, Isai and Wee (2008), and Leang and Minh (2012)). Increase in the returns on crude oil prices lead to stronger economies for oil- exporters and higher production costs for oil- exporters. Hence, it may leads to appreciation of oil- exporter currencies comparatively to those of oil-importers (Duboom and Chims (2019). Studies have also provided evidence on the causality between these two economic indicators runs from oil prices to foreign exchange rate (chandhuri and Aamiel (1998), Ayadi (2005), Chen and Chen (2007), (Liang and Minh, 2012), Krugman (1984) coundert, Mignon and Penot (2007), Ben assy – Querea Migmonb and penot (2007).

Multivariate GARCH (MGARCH) models have proven to be successful in capturing volatility spillovers and co-movements across markets. They are applied in examining whether the volatility of a particular market leads to the volatility of other markets and whether a shock on a market increases the volatility on another market. Though many works on the relationship between exchange rate and crude oil price have been carried out by many researchers with conflicting results and mixed implications to knowledge, in this paper three MGARCH framework- the Baba-Engle-Kraft-Kroner (BEKK)- Generalize Autoregressive Conditional Heteroscedasticity (GARCH) model, the Dynamic Conditional Correlation (DCC)- GARCH model and Constant conditional correlation (CCC)-GARCH model were applied to modeling the two economic indicators separately, and their fitting performance was further compared using the selection criteria, Akaike's Information Criterion (AIC) and Schwarz's Bayesian Information Criterion (SBC).
The paper is organized as follows: The second section provides the review of previous literature. The section three describes the GARCH modeling framework that are employed, and section four gives summary of the statistics and interpretation of result. While section five, provides conclusions.

LITERATURE REVIEW

This section briefly reviews very recent related previous studies, which used the BEKK, DCC and CCC multivariate GARCH-models.
Olson et al., (2014) in a study to model volatility transmission between Goldman Sach's Energy Index and the S&P, used BEKK, CCC, DCC, VIRF he discovered Low S&P 500 returns cause substantial increases in the volatility of the energy index; a weak response from S&P 500 volatility to energy price shocks. Gounopoulos et al. (2013) examined the correlation between stock returns and currency exposures of US, UK, and Japanese banks and insurance companies by using a BEKK model. Long et al. (2014) in a study, analyzed the conditional time-varying currency betas for five developed and six emerging financial markets by using a BEKK model. Using BEKK model, Caporale et al. (2015) tested the impact of exchange rate uncertainty on net equity and net bond flows and on their dynamic linkages. Hoon and Yoon (2013), studied the price returns and volatility linkages between the foreign exchange and stock markets in Korea, using the co-integration test and bi-variate GJR-GARCH (1,1) model based on the BEKK approach. Pelinescu (2014) applied MGARCH-BEKK on the exchange rate for Romanian, Polish, Czech Republic; and found that covariance correlation is higher in the case of the European market (Romanian, Polish and Czech Republic). Ferreira et al., (2014) also used MGARCH-BEKK to model volatility transmission between Brazilian and American stock market. They found evidence of contagion in the indices of Brazil's stock market, increase in the correlation between the indices of the U.S. and Brazilian markets. Bekiros (2014) in a study to model currency and stock markets for firms in Taiwan applied CCC, DCC MGARCH, and BEKK and found ambiguous situation of volatility size effects of the returns to stock prices for large and small firms. Other interesting empirical studies contributions on examining volatility spillovers effects could be found in, Cardona et al. (2017), Bae et al., (2003), Engle (2002), Zhou (2016), Lee (2006), Skintzi and Refenes (2006), Fedorova and Saleem (2010), Olson et al. (2017) etc.

METHODOLOGY

Material and Methods

Data used for this study comprise of ten-year monthly data (2009 – 2018) of crude oil price and exchange rate sourced from the central bank of Nigeria. Weekly data on the Nigerian Naira exchange rate against that of three major currencies; US dollar, European Union's Euro, the British Pound collected from the Central Bank of Nigeria will be used. Data will be transformed before using them in the models because of their volatility. Data analysis in this study was carried out by using the E-View version 10.0 application. In this paper, the multivariate GARCH (MGARCH) models to be considered including BEKK, CCC and DCC models.

Baba–Engle–Kraft–Kroner (BEKK)
A model of the conditional covariance matrix that can be viewed as a restricted version of the VEC model is the Baba-Engle-Kraft-Kroner (BEKK) defined in Engle and Kroner (1995). It has the attractive property that the conditional covariance matrices are positive definite by construction. The first step in the multivariate GARCH methodology is to specify the mean equation. Thus, the mean equation for return series is specified as follows:

$$R_t = \mu_t + \theta R_{i,t-1} + \varepsilon_t, \qquad \varepsilon_t = H_t^{1/2}\eta_t \tag{1}$$

where: $R_t = \left(R_t^S, R_t^B\right)'$ is a vector of returns of the Nigerian stock and Bond markets respectively, θ refers to a 2 x 2 matrix of coefficients, $\varepsilon_t = \left(\varepsilon_t^S, \varepsilon_t^B\right)'$ is the vector of error terms of conditional mean equation for stock and bond markets returns respectively. $\eta_t = \left(\eta_t^S, \eta_t^B\right)'$ is a sequence of independently and identically distributed (i.i.d) random errors; H_t is conditional variance-covariance for both market returns. The second step is specifying the conditional variance-covariance equation. Thus, the BEKK representation of Multivariate GARCH (1,1) model is given by:

$$H_t = CC' + A\varepsilon_{t-1}\varepsilon_t^{'}A' + BH_{t-1}B' \tag{2}$$

Where H_t is the conditional variance matrix, C is an upper triangular 2 ×2 matrix, B is a 2 ×2 square matrices of parameters which depicts the extent to which current levels of conditional variance are related to past conditional variances. A, a 2 ×2 square matrix that measures the extent to which conditional variances are correlated with past square errors. The elements of parameter matrices, A, B and C are express as follows:

$$A = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \qquad B = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \qquad C = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ 0 & \gamma_{22} \end{pmatrix} \tag{3}$$

The significance of the diagonal parameters $\beta_{11}(\beta_{22})$ is evidence of impacts of own past innovations on the current volatility in either the crude oil price or exchange rate and the significance of $\alpha_{11}(\alpha_{22})$ is evidence of influence of past volatilities on current volatility in the stock or bond market. While the significance of the off-diagonal parameters $\beta_{12}(\beta_{21})$ shows evidence of cross-volatility shocks between the crude oil price and exchange rate, and $\alpha_{12}(\alpha_{21})$ is evidence of cross volatility spillover between the two economic indicators considered.

Dynamic Conditional Correlation MGARCH (DCC-MGARCH)

The DCC model, proposed by Engle and Sheppard (2001) and Engle (2002), is a new class of multivariate model, which is particularly well suited to examine correlation dynamics among assets. According to Engle (2002), the Dynamic conditional correlation (DCC) model process can be expressed as

$$H_t = D_t R_t D_t \tag{4}$$

where D_t represents a (k×k) diagonal matrix of the conditional volatility of the returns on each asset in the sample and R_t is the (k×k) conditional correlation matrix. The DCC-GARCH model estimates conditional volatilities and correlations in two steps. In the first step the mean equation of each asset in the sample, nested in a univariate GARCH model of its conditional variance is estimated. Hence, we can define D_t as follows:

$$D_t = \left(h_{iii}^{1/2}, ..., h_{kkt}^{1/2} \right) \tag{5}$$

where: h_{iit}, conditional variance of each indicator, is assumed to follow a univariate GARCH (p_t,q_t) process, given by the following expression:

$$h_{i,t+1} = \theta_i + \sum_{p=1}^{p_i} \xi_{i,p} \varepsilon_{i,t-p}^2 + \sum_{q=1}^{q_i} \psi_{i,q} h_{i,t-q} \tag{6}$$

Moreover, the parameters ξ and ψ must be larger than zero, but the sum has to be less than one. One may note that these are conditions of the univariate GARCH to be stationary, but which is applied in the DCC model. (Orskaug, 2009) . That is

$$\xi_{i,p} > 0; \psi_{i,q} > 0 \quad \text{and} \quad \sum_{p=1}^{p_i} \xi_{i,p} \varepsilon_{i,t-p}^2 + \sum_{q=1}^{q_i} \psi_{i,q} h_{i,t-q} < 1$$

These univariate variance estimates are then used to standardize the zero mean return innovations for each asset. The correlation in the DCC model is then given by

$$Q_t = \left(1 - \xi - \psi\right) \bar{Q} + \xi_i \mu_{t-1} \mu_{r-1}' + \psi Q_{t-1} \tag{7}$$

, $Q_t = \operatorname{cov}\left[\mu_t \mu'_t\right]$ Additionally, the parameters ξ and ψ are scalars and diag(Q) is used to rescale then parts of in order to fulfill that $\left|\rho_{ij}\right| = \left| \frac{q_{\text{int}}}{\sqrt{q_{ii,t} q_{jj,t}}} \right| \leq 1$ and where $\sqrt{q_{nnt}}$ is the content of the matrix .
The conditional correlation co-efficient between two markets I and j are then computed as follows

$$\rho_{12,t} = \frac{\left(1 - \xi - \psi\right) \bar{q}_{12} + \mu_{1,t-1} \mu_{2.t-1} + \psi q_{12,t-1}}{\sqrt{\left[\left(1 - \xi - \psi\right) \bar{q}_{11} + \xi \mu_{1,t-1}^2 + \psi q_{11,t-1}\right]} \sqrt{\left[\left(\left(1 - \xi - \psi\right) \bar{q}_{22} + \xi \mu_{2,t-1}^2 + \psi q_{22,t-1}\right)\right]}} \tag{8}$$

for $\bar{q} = \frac{1}{T} \sum_{t=1}^{T} \mu_t \mu'_t$

3.3 Constant conditional correlation model (CCC)

The first reparametrization of Σ_t is to use the conditional coefficients and variances of ε_t . Specifically, we write as

$$\Sigma_t = \left[\sigma_{ij,t}\right] = D_t \rho_t D_t \tag{9}$$

where ρ_t is the N×N conditional correlation matrix of ε t, and D_t is N ×N diagonal matrix consisting of the conditional standard deviations of elements of ε_t $\left(i.e.\quad D_t = diag\left\{\sqrt{\sigma_{ij,t}}, \ldots\ldots \sigma_{NN,t}\right\}\right)$ Because ρ_t is symmetric with unit diagonal elements, the time evolution of Σ_t is governed by that of the conditional variances $\sigma_{ii,t}$ and the elements $\rho_{ij,t}$ of ρ_t , where j < i and 1< I <N . Therefore, to model the volatility of ε, it suffices to consider the conditional variances and correlation coefficients of it ε (Tsay, 2005). The conditional variances $\sigma_{ii,t}$ are modeled by a univariate GARCH model. Hence,

$$\sigma_{ij,t} = \rho_{ij,t}\sqrt{\sigma_{ii,t}\sigma_{jj,t}} \tag{10}$$

Positivity of S_t follows from the positivity of ρ_t and that of each $\sigma_{ii,t}$ (Bauwens, 2005). As was said before, it is often difficult to verify the condition that the conditional variance matrix of an estimated multivariate GARCH model is positive definite. Furthermore, such conditions are often very difficult to impose during the optimization of the log-likelihood function. However, if we postulate the simple assumption that the correlations are time invariant, these difficulties elegantly disappear (Tse, 2000). Bollerslev (1990) suggested a multivariate GARCH model in which all conditional correlations are constant and the conditional variances are modeled by univariate GARCH models. This is so-called CCC model (constant conditional correlation) (Franke et al., 2005). The constant conditional correlation model (CCC) is defined as

$$\rho_t = \rho = \left[\rho_{ij}\right], \qquad \rho_{ii} = 1 \tag{11}$$

Hence,

$$\sigma_{ij,t} = \rho_{ij,t}\sqrt{\sigma_{ii,t}\sigma_{jj,t}} \qquad \forall i \neq j \tag{12}$$

and the dynamics of the covariances is determined only by the dynamics of the two conditional variances. There are N(N-1)/2 parameters in ρ (Bauwens, 2005). Because of its simplicity, the CCC model has been very popular in empirical applications. A specific member of the group of CCC models is obtained by further constraining the correlations to be zero. This model is denoted as the no correlation (NC) model. Thus, the CCC model is given by

$$\sigma_{ii,t} = C_i + \sum_{h=1}^{p} \phi h_i \varepsilon^2_{t-h,i} + \sum_{h=1}^{q} \omega h_i \sigma_{t-h,i} \qquad i = 1, \ldots\ldots, k \tag{13}$$

$$\sigma_{ij,t} = \rho_{ij}\sqrt{\sigma_{ii,t}\sigma_{jj,t}} \qquad 1 \leq i \leq j \leq k \tag{14}$$

and the NC model is its special case with 0ij ρ = (Tse, Tsui,1999). The restriction that the constant conditional correlations, and thus the conditional covariances, are proportional to the product of the corresponding conditional standard deviations highly reduces the number of unknown parameters and thus simplifies estimation (Bauwens et al., 2006).

RESULTS AND DISCUSSION

Descriptive Statistics

The descriptive measures on the Exchange rate and Crude oil price data were presented in Table 1 below. The result obtained shows that the mean of the series is 200.05 and 80.63 for Exchange and Crude oil price respectively with the median values of 161.98 and 77.55 in that order and the standard deviation of Exchange rate which is 63.12 and that of Crude oil price is 27.31. The result further showed Exchange rate is positively skewed while Crude oil price appeared slightly positively skewed. The kurtosis for both exchange rate and crude oil were less than 3 as indicated in all the models, therefore, the relatively large kurtosis. The probability value (p-value) of both indicators was significant at 5% which indicated that the data are not normally distributed.

Table 1: Summary Statistic of the Nigerian Exchange rate and Crude oil price

	EXCHANGE RATE	CRUDE OIL PRICE
Mean	200.0537	80.63058
Median	161.9750	77.55000
Maximum	309.7300	128.0000
Minimum	146.5900	30.66000
Std. Dev.	63.12353	27.30687
Skewness	0.976804	0.058352
Kurtosis	2.142713	1.594247
Jarque-Bera	22.75763	9.948806
Probability	0.000011	0.006913
Sum	24006.44	9675.670
Sum Sq. Dev.	474165.1	88734.17
Observations	120	120

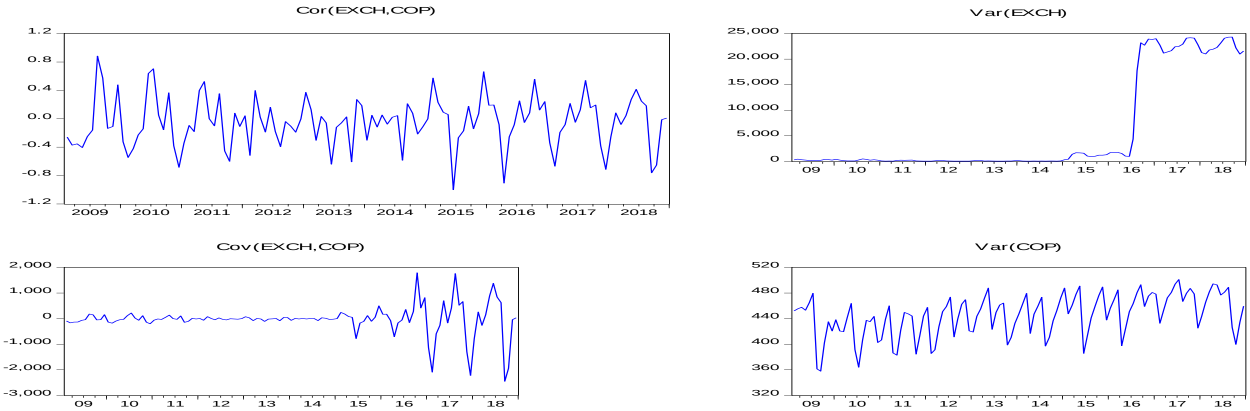
Table 2 below shows that the result of the correlation value between EXCH and COP is -0.006, indicating a negatively weak relationship. Base on the result, an increase in COP will have a slid negative impact on the Nigerian exchange rate.

Table 2: Correlation between EXCH and COP

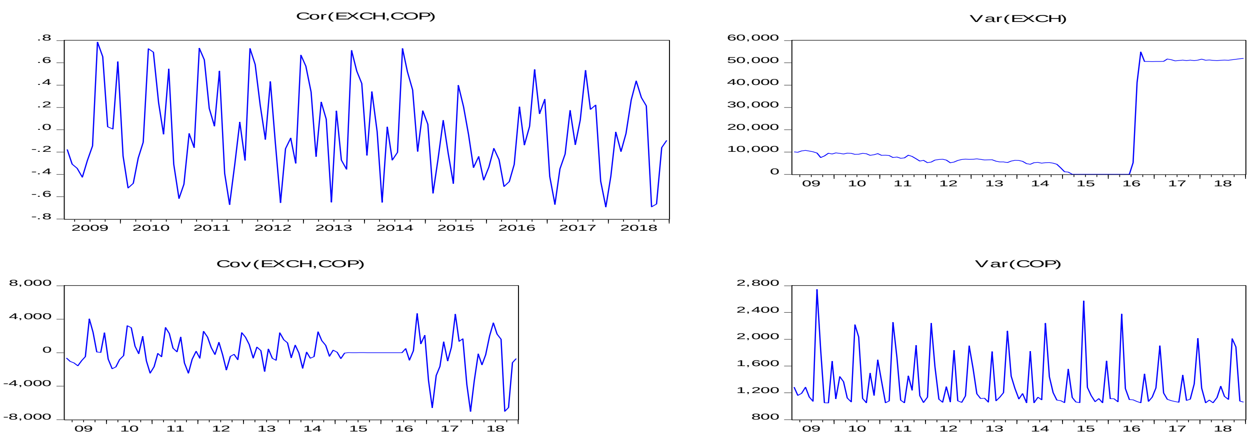
	EXCH	COP
EXCH	1	
COP	-0.006	1

A look at figure 1 confirms the negatively weak relationship between crude oil price and Nigerian exchange rate. It is evident from the behaviour of conditional covariances (Figure 1) that correlation between crude oil price and exchange rate is very unstable over time. The conditional variance has been relatively stable, except the second half of 2008 and the year 2009, due to the financial turbulence. Some higher degree of conditional variance can be spotted in the beginning of the sample for EXCH given the COP. The graphs confirm what the received values of B indicate, that the conditional variance is persistent. However, we can clearly see quite similar dependence in conditional volatilities for each series. The next graph visualizes the estimated conditional correlation: The estimated conditional covariance graph shows a more non stable pattern, although it seems that the moving pattern is centralized around an upward rising trend, correlating from around -0.8 to about 0.8 for BEKK model, -0.6 to 0.8 for DCC and -1.0 to 0.8 for CCC models. In general, CCC graph are more volatile than the other multivariate models. The general impression in all the models is that the movement of EXCH seems to be trending upwards during this ten-year period especially since 2015. Furthermore, another impression is that the graph shows tendencies of correlational drops during periods of economic crises, most easily observed is the period around the year 2009.

BEKK Model



DCC Model



CCC Model

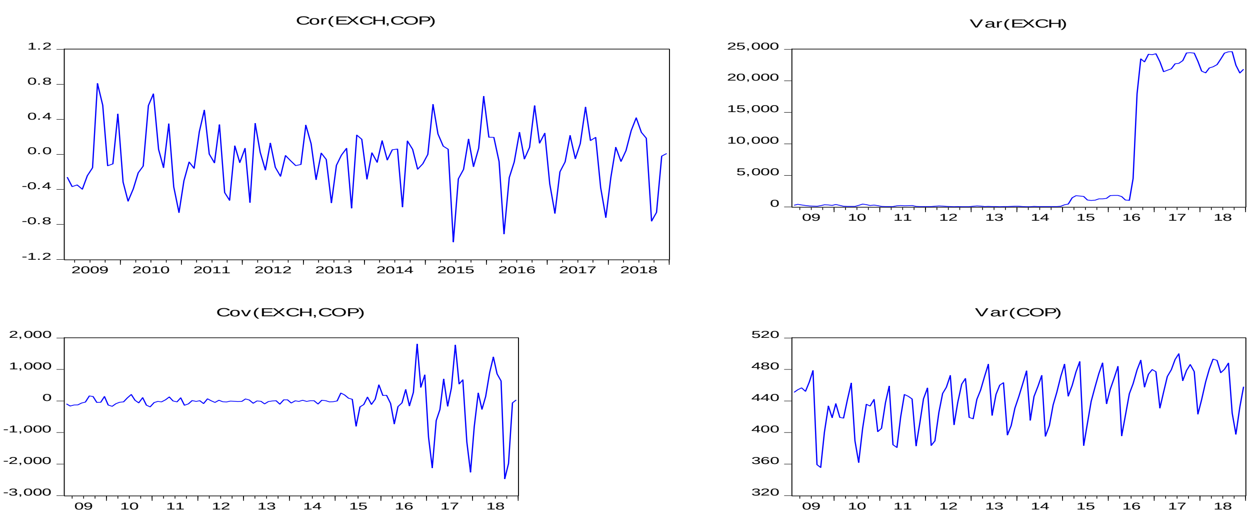


Figure 1. The graphs of correlation of COP and EXCH, variance of COP and EXCH, and covariance of COP and EXCH respectively.

DISCUSSION OF RESULTS

MGARCH (1,1)-BEKK Results

We start with the BEKK (p,q) model. Estimated coefficients of the parameters of the BEKK model for both data samples Nigerian exchange rates and crude oil price indices respectively can be found in Tables 3 below. The order of the model was estimated as BEKK (1,1) with K = 1 and the method for the estimation of parameters was maximum log-likelihood.

Table 3 : Estimated Results of the GARCH-BEKK Model for EXCH and COP

Parameter	Coefficient	Std. Error	z-Statistic	Prob.
γ_{11}	26.77444	20.22682	1.323710	0.1856
γ_{12}	-11.67113	27.13721	-0.430078	0.6671
γ_{22}	139.8192	78.25289	1.786761	0.0740
β_{11}	1.000292	0.527850	1.895031	0.0581
β_{12}	0.431181	0.206598	2.087052	0.0369*
β_{22}	-0.050821	0.120709	-0.421022	0.6737
α_{11}	0.099439	0.337259	0.294846	0.7681
α_{12}	-0.019246	0.375931	-0.051197	0.9592
α_{22}	0.736694	0.094522	7.793922	0.0000*

Notes. α_{ii} *and* β_{ii} *are the corresponding ARCH and GARCH parameters for each indicator . *Significant at 5% level; **significant at both 1% and 5% levels.*

Notice from Table 3 that the estimates of all the diagonal parameters, β_{12} only is significant at 5% significant level, indicating that the own past shocks affect the current volatility of the Nigeria exchange rate However, α_{11} and β_{22} are all not statistically significant at both 5% and 1% significant levels suggesting that own past volatility does not affect the current volatility of the crude oil price and exchange rate in Nigeria.

From the off-diagonal element, β_{12} is significant at 5% significance level, showing evidence of bi-directional shock transmissions between the Nigerian exchange rate and crude oil price.

MGARCH DCC (1,1) Results

To confirm the superiority of DCC model over the BEKK model and the CCC model, a test of dynamic correlation is made in order to see which one of the models that is most suitable for the data. A p-value of less than 0.05 is received, pointing out the absence of constant correlation and the data is thus suitable for using a DCC model compared to the CCC model where the correlation is assumed to be constant. Consequently, an estimation of the DCC model will be done. The table below consists of the estimation results:

Table 4: Bivariate DCC (1,1) Estimation Results between the Nigerian exchange rate and Crude oil price

Parameter	Coefficient	Std. Error	z-Statistic	Prob.
θ_{11}	0.000224	0.000237	0.945170	0.3446
θ_{12}	-0.119764	0.521052	-0.229850	0.8182
θ_{22}	1050.230	885.4214	1.186135	0.2356
ξ_{11}	2.078189	0.647107	3.211506	0.0013*
ξ_{22}	0.784453	0.465242	1.686119	0.0918
ψ_{11}	-1.58E-06	513.3339	-3.07E-09	1.0000
ψ_{22}	-0.000131	2474.051	-5.28E-08	1.0000

Table 4 reports the results of estimating the complete MGARCH DCC model for Nigerian exchange rate and Crude oil price. The bivariate DCC model applied in the analysis allows for a time varying correlation structure. The coefficient θ corresponds to the mean equation parameter, while ξ and ψ represents the conditional variance of EXCH versus COP. As reflected in the Table 4, all parameters are positive except for ξ_1 and ξ_2. Only one parameter was found to be significant at either 5% level of significance. The insignificance of mean equation parameter θ shows the independence of returns on their lag returns. Furthermore, the volatility persistence in these markets is measured by $(\xi+\psi)$, and looking at Table 4, the sums of the variance equation parameters ξ_{ii} and ψ_{ii} are greater than 1, indicating rather strong persistence in conditional variances and the GARCH model is nonstationary and the volatility will eventually explode to infinity (Banerjee and Sarkar, 2006). The estimated conditional correlation is negative (-0.000131) and very weak, reflecting a weak negative relationship between the crude oil price and the Nigeria exchange rate. This suggests that the oil price and exchange rate volatility tend to move in opposite directions.

4.2.3. MGARCH CCC (1,1) Results

Table 5: Bivariate CCC (1,1) Estimation Results between the Nigerian exchange rate and Crude oil price

Parameter	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
γ_{11}	35.75548	28.06577	1.273989	0.2027
γ_{12}	-11.38056	28.42096	-0.400429	0.6888
γ_{22}	139.6957	122.6444	1.139030	0.2547
ϕ_{11}	0.998309	0.558217	1.788389	0.0737
ϕ_{12}	0.430984	0.221488	1.945854	0.0517
ϕ_{22}	-0.051079	0.134666	-0.379303	0.7045
ω_{11}	0.100220	0.376377	0.266277	0.7900
ω_{12}	-0.009435	0.403405	-0.023389	0.9813
ω_{22}	0.736390	0.180030	4.090384	0.0000*

The CCC estimates of the conditional correlations between the volatilities of crude oil price and the Nigerian Exchange rate and also estimates of the GARCH parameters are presented in Table 5. To interpret this, ϕ high ω means that the conditional variance is persistent. A high ϕ means that the volatility is spiky. Since our values of ω are low, the conditional variances seem to be less persistent. As the estimates of both ϕ, the impact of past shocks on current conditional correlations, and ω the impact of previous dynamic conditional correlations, are statistically not significant at 5% levels, this clearly indicates that the conditional correlations are constant. As can be seen $\phi_{11} < \phi_{22}$, indicating a less spiky time series. The estimate of ω_{11} is generally low and close to zero, whereas the estimate ϕ_{11} is extremely high and close to unity. The conditional correlations between the indices are dynamic. It can be observed that the non-diagonal elements are non significant which mirrors the theoretical fact that these parameters often are redundant.

4.3. Model Adequacy Checking and Model Selection

We compared the two models applied according to their goodness-of-fit statistics, namely Akaike's Information Criterion (AIC) and Schwarz's Bayesian Information Criterion (SBC) and H (HQIC) as shown in the table below and the BEKK model was indeed the best performer, showing the smallest values for all criteria.

Table 6: Comparing the Four variables under the Scalar BEKK (1, 1) model

	BEKK Model	DCC model	CCC model
Log-Likelihood	-1055.36	-1094.34	-1055.87
AIC	17.97	18.59	17.98
SIC	18.20	18.87	18.31
HQIC	18.11	18.71	18.11

Log L (Log-Likelihood), Akaike Information Criteria (AIC), Schwarz Information Criteria (SIC), Hannan-Quim Information Criteria (HQIC)

5. CONCLUSIONS

This paper investigated the volatility transmission between the Nigerian exchange rate and crude oil price from January 2009, to December 2018. using three multivariate GARCH Models (BEKK, DCC, and CCC). The descriptive statistics showed non- normality between exchange rate and crude oil price. The estimated conditional covariance graph shows a more non stable pattern, although it seems that the moving pattern is centralized around an upward rising trend, correlating from around -0.8 to about 0.8 for BEKK model, -0.6 to 0.8 for DCC and -1.0 to 0.8 for CCC models. In general, CCC graph are more volatile than the other multivariate models. The general impression in all the models is that the movement of EXCH seems to be trending upwards during this ten-year period especially since 2015. As the estimates of both the impact of past shocks on current conditional correlations, the impact of previous dynamic conditional correlations, are statistically not significant, which clearly indicates that the conditional correlations are slightly constant. The conditional correlations between the indices are dynamic. All analyzed models (BEKK, DCC, and CCC) show similar behavior in variances and covariances. In particular, the estimated conditional correlation with DCC model is negative and very weak, reflecting a weak negative relationship between the crude oil price and the Nigeria exchange rate. This suggests that the oil price and exchange rate volatility tend to move in opposite directions. The main finding is that conditional covariances exhibit significant changes over time for both exchange rate and crude oil price. The performance evaluation carried out revealed that Bivariate GARCH-BEKK model effectively captures the volatility spillovers between crude oil price and exchange rate in Nigeria. However, the study observed that in general, CCC graph are more volatile than the other multivariate models. The BEKK showed a better ability of forecasting, since the DCC tends to underestimate the conditional correlation. Thus, we conclude that there is independence as well as volatility spillover between the returns on exchange rate and crude oil price.

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