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On the Area Biased Quasi-Transmuted Uniform Distribution

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ABSTRACT
This study considered a new Uniform Distribution, the Area Biased Quasi-Transmuted Uniform distribution. Several essential properties of the new distribution were derived. The simulation results show that that the Area Biased Quasi-Transmuted Uniform distribution fitted well and therefore has potentials in modeling observations with a restricted range.
Keywords: Area biased quasi-transmuted uniform distribution, Odd function, Order statistics, Hazard function, Cumulative hazard function, Assymptotic behaviour

INTRODUCTION
Fitting distributions to data based on generalized distributions derived from baseline distributions has always being the interest of researchers in distribution modeling. Existing generalized distributions depend on their methods of generation. Some examples of existing generalized distributions in literature include Length Biased Quasi-Transmuted Uniform distribution by Osowole and Onyeze (2020); Generalized Length Biased Exponential distribution by Maxwell *et. al.* (2019); Modified Uniform distribution by Osowole and Ayoola (2019); Modified Complementary Mukherjee-Islam distribution by Osowole (2019); Size-Biased Ailamujia distribution by Rather *et. al.* (2018); Generalized Size-Biased Poisson-Lindley distribution by Shanker and Shukla (2018); Transmuted Mukherjee-Islam distribution by Rather and Subramanian (2018); Weighted Pareto Type II distribution by Para and Jan (2018); Size-Biased Lindley distribution by Ayesha (2017); Size-Biased Poisson-Shanker distribution by Shankar (2017); Weighted Ailamujia distribution by Uzma *et. al.* (2017); Weighted Lindley distribution by Asgharzadeh *et. al.* (2016); Weighted Exponential distribution by Mizaal (2015); Length-Biased Weighted Maxwell distribution by Modi (2015); Exponentiated Weighted Exponential distribution by Oguntunde (2015); Weighted Feller-Pareto distribution by Odubote and Oluyede (2014); Exponentiated Weighted Weibull distribution by Idowu and Bamiduro (2014); Double Weighted Rayleigh distribution by Ahmad and Ahmad (2014); Length-Biased Exponentiated Inverted Weibull distribution by Seenoi *et. al.* (2014); and Weighted Inverse Weibull distribution by Sherina and Oluyede (2014). A practical need for generalized distributions was mooted by Subramanian and Rather (2018). They noted that generalized distributions are needful in situations where existing conventional distributions failed.

This study however considers the Area Biased Quasi-Transmuted Uniform Distribution (ABQTUD) by drawing inspiration from the previous contributions to knowledge of Shankar (2017); Uzma *et. al.* (2017) and Rather *et. al.* (2018).

The Area Biased Quasi-Transmuted Uniform Distribution

The probability density function (pdf) of the Quasi-Transmuted distribution according to Osowole and Ayoola (2019) is defined as

$$g^*(x) = 2ax + (1-a), 0 < x < 1; a \in (0, \frac{1}{2}, 1) \quad \text{……(1.0)}$$

with cumulative distribution function (cdf) defined as

$$G^*(x) = ax^2 + (1-a)x, 0 < x < 1; a \in (0, \frac{1}{2}, 1) \quad \text{……(2.0)}$$

A length biased distribution according to Subramanian and Rather (2018) is a weighted distribution $f_w(x)$ defined as

$$f_w(x) = \frac{w(x) f(x)}{E[w(x)]}, x > 0 \quad \text{……(3.0)}$$

where w(x) is non negative such that $E[w(x)] = \int_{-\infty}^{+\infty} w(x) f(x) dx < \infty$

We note that (3.0) becomes an area biased distribution when w(x) is chosen such that w(x) = x^2 and it becomes a size biased distribution when w(x) is chosen such that w(x) = x^c. Combining (1.0) and (3.0), the probability density function of the Area biased quasi transmuted uniform distribution is given as

$$g_{ABQTUD}(x) = \frac{x^2[2ax + (1-a)]}{E[x^2]}, 0 < x < 1; a \in (0, \frac{1}{2}, 1)$$

$$= \frac{2ax^3 + (1-a)x^2}{[(a+2)/6]}$$

$$= \frac{12ax^3 + 6(1-a)x^2}{[(a+2)]}, 0 < x < 1; a \in (0, \frac{1}{2}, 1) \quad \text{……(4.0)}$$

(4.0) shall henceforth be referred to in this study as the Area Biased Quasi-Transmuted Uniform Distribution (ABQTUD). The cumulative distribution function of (4.0) is defined as

$$G_{ABQTUD}(x) = \int_0^x g_{ABQTUD}(t)dt$$

$$= \int_0^x \frac{12at^3 + 6(1-a)t^2}{a+2} dt$$

$$= \frac{3ax^4 + 2(1-a)x^3}{a+2}, 0 < x < 1; a \in (0, \frac{1}{2}, 1) \qquad \text{…………………………………………(5.0)}$$

Some Characterizations of the Area Biased Quasi-Transmuted Uniform Distribution Moments

The k[th] raw moment for the Area Biased Quasi-Transmuted Uniform distribution is

$$\mu_k^1 = E(X^K) = \int_0^1 x^k \; g_{ABQTUD}(x)\, dx$$

$$= \frac{1}{a+2}\int_0^1 x^k [12ax^3 + 6(1-a)x^2]dx$$

$$= \frac{1}{a+2}[\frac{12ax^{k+4}}{k+4} + \frac{6(1-a)x^{k+3}}{k+3}]\Big|_0^1$$

$$= \frac{1}{a+2}\left[\frac{12a(k+3) + 6(k+4)(1-a)}{(k+4)(k+3)}\right]$$

$$= \frac{1}{a+2}\left[\frac{6a(k+2) + 6(k+4)}{(k+4)(k+3)}\right], a \in (0, \frac{1}{2}, 1)$$

Specifically,

$$\mu_1^1 = \mathrm{E}(X) = mean = \frac{1}{a+2}\left[\frac{9a+15}{10}\right]$$

$$= \frac{3}{4}, \frac{19.5}{25} \& \frac{4}{5} \; for \; a \; = \; 0, \frac{1}{2} and \;\; 1 \; respectively$$

Also,

$$\mu_2^1 = \frac{1}{a+2}\left[\frac{4a+6}{5}\right]$$

$$= \frac{3}{5}, \frac{8}{12.5} \& \frac{2}{3} \;\; for \;\; a = 0, \frac{1}{2} \& 1 \; respectively$$

From the

μ_1^1 *and* μ_2^1 above, the variance of X, $V(X) = E(X-\mu)^2 = \mu_2 = \mu_2^1 - (\mu_1^1)^2$ That is, $\sigma_X^2 = 0.0375, 0.0316 \,\&\, 0.0267$ for a = $0, \frac{1}{2} \& 1$ respectively. The variance can be seen to reduce as the value of a is increasing. This is the expected pattern for an ideal generalized distribution.

We note also that:

$$\mu_3^1 = E(X^3) = \frac{1}{a+2}\left[\frac{5a+7}{7}\right]$$

$$= \frac{1}{2}, \frac{19}{35} \& \frac{4}{7} \text{ for } a = 0, \frac{1}{2} \& 1 \text{ respectively}$$

$$\mu_3 = E(X-\mu)^3 = \mu_3^1 - 3\mu\mu_2^1 + 2\mu^3$$

$$= -0.00625, -0.00564 \text{ and } -0.00457 \text{ for } a = 0, \frac{1}{2} \& 1 \text{ respectively}$$

The implies that the coefficient of skewness, $\alpha_3 = \frac{\mu_3}{\sigma^3}$. That is

$\alpha_3 = -0.86066, -1.00403$ and -1.04749 respectively for a =$0, \frac{1}{2} \& 1$. The skewness can be seen to be reducing as the value of a is increasing. This again is the expected pattern for an ideal generalized distribution.

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Furthermore,

$$\mu_4^1 = E(X^4) = \frac{1}{a+2}\left[\frac{9a+12}{14}\right]$$

$$= \frac{3}{7}, \frac{16.5}{35} \& \frac{1}{2} \text{ for } a = 0, \frac{1}{2} \& 1 \text{ respectively}$$

$$\mu_4 = E(X-\mu)^4 = \mu_4^1 - 4\mu\mu_3^1 + 6\mu^2\mu_2^1 - 3\mu^4$$

$$= 0.00435, 0.00352 \text{ and } 0.00263 \text{ for } a = 0, \frac{1}{2} \& 1 \text{ respectively}$$

This implies that the coefficient of kurtosis, $\alpha_4 = \frac{\mu_4}{\sigma^4}$. That is

$\alpha_4 = 3.0955, 3.52410$ and 3.68781 respectively for a = $0, \frac{1}{2} \& 1$. The coefficient of kurtosis can be seen to be increasing as the value of a increases from 0 to 1. This indicates that the Area Biased Quasi-Transmuted distribution being considered has the potentials of a generalized distribution.

Coefficients of Variation and Dispersion

The coefficients of variation and dispersion for the Area Biased Quasi-Transmuted distribution are $CV = \frac{\sigma}{E(X)}$ *and* $CD = \frac{\sigma^2}{E(X)}$. Specifically,

$$CV = \frac{\sqrt{\frac{-a^2+10a+15}{100(a+2)^2}}}{\left(\frac{9a+15}{10a+20}\right)} = 0.25820, 0.23077, 0.20412 \text{ for a} = 0, \frac{1}{2} \& 1$$

and $$CD = \frac{-a^2+2a+5}{10(a+3)(a+2)} = 0.0500, 0.04051, 0.03333 \text{ for a} = 0, \frac{1}{2} \& 1$$

The two coefficients above can be seen to be reducing as the value of a is increasing. Also, the coefficients of variation fall within the acceptable limit with the least CV obtained for a = 1. This indicates that the Area Biased Quasi-Transmuted distribution is appropriate as a generalized distribution.

Moment Generating Function

The moment generating function for the Area Biased Quasi-Transmuted Uniform distribution is defined as

$$M_X(t) = \mathrm{E}(e^{tX}) = \int_0^1 e^{tX} g_{ABQTUD}(x)dx$$

$$= \int_0^1 \left[1 + tX + \frac{(tX)^2}{2!} + \ldots\right] g_{ABQTUD}(x)dx$$

$$= \int_0^1 \sum_{j=0}^{\infty} \frac{t^j}{j!} X^j g_{ABQTUD}(x)dx$$

$$= \sum_{j=0}^{\infty} \frac{t^j}{j!} \mu_j^1$$

$$= \frac{1}{a+2} \sum_{j=0}^{\infty} \frac{t^j}{j!} \left[\frac{6a(j+2)+6(j+4)}{(j+3)(j+4)}\right], \text{ a } \varepsilon \ (0, \frac{1}{2}, 1)$$

Characteristic Function
The characteristic function for the Area Biased Quasi-Transmuted Uniform distribution is defined as

$$\phi_x(t) = M_X(it)$$
$$= \mathrm{E}(e^{itX})$$
$$= \int_0^1 e^{itX} g_{ABQTUD}(x)dx$$
$$= \frac{1}{a+2}\sum_{J=0}^{\infty}\frac{(it)^j}{J!}\left[\frac{6a(j+2)+6(j+4)}{(j+3)(j+4)}\right], \text{ a } \varepsilon \ (0,\frac{1}{2},1)$$

Cumulant Generating Function
The cumulant generating function of a random variable X from the Area Biased Quasi-Transmuted Uniform distribution is defined as

$$K_X(t) = In\left[M_X(t)\right]$$
$$= In\left[\sum_{J=o}^{\infty}\frac{t^j}{j!}\mu_j^1\right]$$
$$= In\left[\frac{1}{a+2}\sum_{J=0}^{\infty}\frac{t^j}{j!}\left[\frac{6a(j+2)+6(j+4)}{(j+3)(j+4)}\right]\right], \text{ a } \varepsilon \ (0,\frac{1}{2},1)$$

Hazard Function
The hazard function for the Area Biased Quasi-Transmuted Uniform distribution is defined as

$$h(x) = \frac{g_{ABQTUD}(x)}{1-G_{ABQTUD}(x)}$$
$$= \frac{12ax^3+6(1-a)x^2}{a+2-[3ax^4+2(1-a)x^3]}, \text{a } \varepsilon \ (0,\frac{1}{2},1)$$

Reverse Hazard Function
The reserve hazard function for the Area Biased Quasi-Transmuted Uniform distribution is defined as

$$h_r(x) = \frac{g_{ABQTUD}(x)}{G_{ABQTUD}(x)}$$
$$= \frac{12ax^3+6(1-a)x^2}{3ax^4+2(1-a)x^3}, \ a \in (0,\frac{1}{2},1)$$

Survival Function

The survival function for the Area Biased Quasi-Transmuted Uniform distribution is defined as

$$S_X(x) = 1 - G_{ABQTUD}(x)$$
$$= \frac{a+2-[3ax^4+2(1-a)x^3]}{a+2}, a \in (0, \frac{1}{2}, 1)$$

Asymptotic Behaviour

The behavior of the Area Biased Quasi-Transmuted Uniform Distribution (ABQTUD) (4.0) is now assessed as x → 0 and x → ∞.

Thus,

$$\lim_{x\to 0}\left[g_{ABQTUD}(x)\right] = \lim_{x\to 0}\left[\frac{12ax^3+6(1-a)x^2}{a+2}\right] = 0 \text{ and } \lim_{x\to\infty}\left[g_{ABQTUD}(x)\right] = \lim_{x\to\infty}\left[\frac{12ax^3+6(1-a)x^2}{a+2}\right] = 3, 3.6 \,\&\, 4 \text{ for a=0,} \frac{1}{2} \,\&\, 1$$

Odd Function

The odd function of the Area Biased Quasi-Transmuted Uniform distribution is

$$ODD_X(x) = \frac{a+2-[3ax^4+2(1-a)x^3]}{3ax^4+2(1-a)x^3}, a \in (0, \frac{1}{2}, 1)$$

Cumulative Hazard Function

The cumulative hazard function for the Area Biased Quasi-Transmuted Uniform distribution is

$$H(x) = Log\left[\frac{a+2-[3ax^4+2(1-a)x^3]}{a+2}\right], a \in (0, \frac{1}{2}, 1)$$

Order Statistics

Let $X_{(1)}$, $X_{(2)}$,…, $X_{(n)}$ be order statistics from the random sample X_1, X_2, …., X_n from the Area Biased Quasi-Transmuted Uniform distribution with $g_{ABQTUD}(x)$ and $G_{ABQTUD}(x)$, the r[th] order statistic where $1 \le r \le n$ is given as

$$h_{(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\, g_{ABQTUD}(x)\left[G_{ABQTUD}(x)\right]^{r-1}\left[1 - G_{ABQTUD}(x)\right]^{n-r}$$

By setting r = n and r =1 in the function ($h_{(r)}(x)$) above, we have the distributions for the largest and lowest order statistics. For the largest order statistic we have that

$$h_{(r=n)}(x) = n\left[\frac{12ax^3+6(1-a)x^2}{a+2}\right]\left[\frac{3ax^4+2(1-a)x^3}{a+2}\right]$$

$$=\frac{n}{(a+2)^n}\left[12ax^3+6(1-a)x^2\right]\left[3ax^4+2(1-a)x^3\right]^{n-1}, a\in(0,\frac{1}{2},1)$$

For the lowest order statistic, we have that

$$h_{(r=1)}(x) = n\left[1-G_{ABQTUD}(x)\right]^{n-1} g_{ABQTUD}(x)$$

$$=n\left[\frac{a+2-[3ax^4+2(1-a)x^3]}{a+2}\right]^{n-1}\left[\frac{12ax^3+6(1-a)x^2}{a+2}\right]$$

$$=\frac{n}{(a+2)^n}\left[a+2-[3ax^4+2(1-a)x^3]\right]^{n-1}\left[12ax^3+6(1-a)x^2\right], a\in(0,\frac{1}{2},1)$$

Random Number Generation

Random numbers can be generated for the Area Biased Quasi-Transmuted Uniform distribution using the quantile function as follows:

:

$$Let\ \frac{1}{a+2}\left[3ax^4+2(1-a)x^3\right] = u \qquad so\ that$$

$$\frac{1}{a+2}\left[3ax^4+2(1-a)x^3\right]-u = 0 \Rightarrow 3ax^4+2(1-a)x^3-u(a+2) = 0$$

where U is a random variable from the Uniform (0,1). *By setting* $a=0$, $3ax^4+2(1-a)x^3-u(a+2)=0$ *becomes* $x^3=u \Rightarrow x=+u^{\frac{1}{3}}$, *where* $U \sim Uniform(0,1)$. By setting a=$\frac{1}{2}$, $3ax^4+2(1-a)x^3-u(a+2)=0$ becomes $1.5x^4+x^3$-2.5u=0. It should be noted that the root of this equation can not be determined as it is.

By setting a=1, $3ax^4+2(1-a)x^3-u(a+2)=0$ becomes $3x^4-3u=0 \Rightarrow x=+u^{\frac{1}{4}}$ where U~Uniform(0,1). The implication here is that for the random number generation, a must be set to 0 or 1.

Simulation Study

The simulation results from the Area Biased Quasi-Transmuted Uniform distribution at a=0 and 1 for sample sizes 1000, 2000 and 5000 respectively are given below (Tables 1.0a and 1.0b)

Table 1.0a: Simulation from the ABQTUD

a =0			a=1		
Sample Size	**Summary Statistics**	**Estimate**	**Sample Size**	**Summary Statistics**	**Estimate**
1000	mean	0.746	500	mean	0.764
	variance	0.0407		variance	0.033
	standard deviation	0.2016		standard deviation	0.181
	skewness	-0.9013		skewness	-0.842
	kurtosis	3.1817		kurtosis	3.071
	coefficient of variation	0.2702		coefficient of variation	0.2151
	coefficient of dispersion	0.0546		coefficient of dispersion	0.0368
2000	mean	0.7467	2000	mean	0.7971
	variance	0.0381		variance	0.0273
	standard deviation	0.1951		standard deviation	0.1653
	skewness	-0.9224		skewness	-0.909
	kurtosis	3.2038		kurtosis	3.256
	coefficient of variation	0.2613		coefficient of variation	0.2074
	coefficient of dispersion	0.0510		coefficient of dispersion	0.0343

Table 1.0b: Simulation from the ABQTUD contd.

a =0			a=1		
Sample Size	**Summary Statistics**	**Estimate**	**Sample Size**	**Summary Statistics**	**Estimate**
5000	mean	0.7520	5000	mean	0.8018
	variance	0.0366		variance	0.0260
	standard deviation	0.1914		standard deviation	0.1612
	skewness	-0.8686		skewness	-1.0537
	kurtosis	3.1026		kurtosis	3.6988
	coefficient of variation	0.2545		coefficient of variation	0.2011
	coefficient of dispersion	0.0487		coefficient of dispersion	0.0324

The simulation results in Table (1.0)a indicate an agreement between the simulated results and the derived theoretical results; especially for the following descriptive statistics: mean, variance, standard deviation, coefficient of variation and coefficient of dispersion. This is a validation of the explored characterizations of the newly proposed Area Biased Quasi-Transmuted Uniform Distribution.

Table 2.0a: Estimates of -InL, AIC, AICC and BICC for comparison between models

Parent Distribution (Uniform (0 , 1))	Comparison Criteria			
	-InL	AIC	AICC	BICC
	0	4.000	4.600	6.271

Table 2.0b: Estimates of -InL, AIC, AICC and BICC for comparison between models

	a = 0				a = 1			
	Comparison Criteria				Comparison Criteria			
	-InL	AIC	AICC	BICC	-InL	AIC	AICC	BICC
ABQTUD	-2.2700	1.4603	2.7235	6.2710	-6.0117	-6.0235	-4.7603	-2.6170

Tables (2.0)a and (2.0)b give the estimates of the Akaike Information Criterion (AIC), Corrected Akaike Information Criterion (AICC) and Bayesian Information Criterion (BIC). These estimates indicate a better fit based on their lower values. The tables show that the Area Biased Quasi-Transmuted Distribution fitted better than the baseline parent distribution (Uniform Distribution (0, 1)). This is similar to the findings observed by Rather *et. al*. (2018) and Uzma *et. al*.(2018). This validates the expected fitting potentials of the Area Biased Quasi-Transmuted Uniform Distribution.

CONCLUSION

This study aimed to propose the Area Biased Quasi-Transmuted Uniform distribution and to consider some of its essential properties including moments, hazard function, reverse hazard function, odd function and cumulative hazard function. These were successfully derived. The simulation study provided an empirical application of the new distribution and further showed that the new distribution performed better than its parent baseline distribution. It is hoped that the Area Biased Quasi-Transmuted Uniform Distribution (ABQTUD) will be attractive to researchers having their observations in the open interval (0, 1).

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