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Conceptual and Procedural Knowledge of Pre-service Teachers in Geometry

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ABSTRACT
This study investigated pre-service mathematics teachers' conceptual and procedural knowledge. The purpose was to find out whether their knowledge of concepts and procedures in mathematics relate to one another. Twenty eight pre-service mathematics teachers were involved as respondents to the study. These students responded to thirty test items, fifteen requiring knowledge of concepts and fifteen requiring knowledge of procedures, both in geometry. The test items are open-ended, and the participants responded to them with no time limit. The findings indicated that the pre-service mathematics teachers possess some knowledge of geometry, in terms of conceptual and procedural. It was also revealed that there is a significant relationship between pre-service mathematics teachers' conceptual knowledge and procedural knowledge in geometry. However, it was shown that performance on conceptual items was higher than on procedural items. It was suggested that attention be paid to both aspects of knowledge, as expertise in mathematics requires both knowledge.

Keywords: Conceptual knowledge; Procedural knowledge; Pre-service teachers; Geometry

1.0 INTRODUCTION
Competence in mathematics requires the knowledge of concepts and procedures. The possession of subject matter knowledge is necessary for a mathematics teacher to be competent and effective in teaching. This is because subject matter knowledge is a combination of the knowledge of both concepts and procedures. Researchers such as Rittle-Johnson & Schneider (2012) have argued that the relation between the two types of knowledge is bi-directional, so ideally, mathematics teachers are expected to demonstrate knowledge of both concepts and procedures. According to Rittle-Johnson and Schneider (2012), the knowledge of one supports the other and vice versa. Similarly, Star (2005) pointed out that it is difficult to differentiate conceptual and procedural knowledge because of their similarity in many respects. He said although researchers often discuss them as separate entities, they rely on each other to develop in mathematics.

Dictionary.com (2015) describes a concept as an idea of something formed by mentally combining all its characteristics or particulars. Conceptual knowledge, which is knowledge of concepts, has to do with abstraction and generalization of particular instances. Having conceptual knowledge is more than just knowing definitions and rules in mathematics. Students may recall certain definitions, rules and procedures, but it cannot be said they possess conceptual understanding. Ability to justify why a statement in mathematics is true or where a mathematical rule comes from is a demonstration of conceptual understanding. For instance, Star, Caronongan, Foegen, Furgeson, Keating, Larson, Lyskawa, McCallum, Porah and Zbiek (2015) pointed out that "there is a difference between a student who can summon a mnemonic device to expand a product such as (a+b)(x+y), and a student who can explain where a mnemonic comes from". Therefore, the ability to recall into memory either a mathematical rule, definition, or procedure and to apply such is not enough justification of having conceptual knowledge. Being able to explain the rule, definition or procedure involved is required for conceptual knowledge evidence. This is because the knowledge of concepts involves understanding of meaning, and not just ability to recall definitions, rules or procedures.

The ability to commit into memory that two negative numbers can result into a positive number when multiplied or divided, is not the same as understanding the reason for the product or quotient to be a positive number. The ability to explain the reason for the answer to be positive is evidence of possessing conceptual knowledge. This knowledge is not restricted to particular problem type (Rittle-Johnson & Schneider, 2012), it can be explicit or implicit since it is not necessarily expressed in words. Kilpatrick, Swafford and Findell (2001) referred to this knowledge as "comprehension of mathematical concepts, operations and relations", (p. 5). Schneider and Stern (2010a) pointed out that conceptual knowledge is the knowledge that provides an abstract understanding of the principles and relations among bits of knowledge in a certain domain. This definition is corroborated by the definition given by Khashan (2014), who defined conceptual knowledge as, "an abstract knowledge addressing the essence of mathematical principles and relations among them", (p. 182).

Furthermore, Isleyen and Isik (2003) described conceptual knowledge in mathematics as the knowledge that comprises of symbols and demonstrations. It means therefore, that this knowledge represents mathematical concepts and relates pieces of mathematical knowledge to each other to provide understanding of concepts and mathematical rules and propositions. Similarly, Baroody, Feil and Johnson (2007) described conceptual knowledge as knowledge of concepts and principles, and their relationship to each other. Therefore, in this study, having conceptual knowledge in mathematics refers to understanding relationships among the concepts, definitions and rules of mathematics, and being able to give satisfactory or justifiable explanations about them. In other words, knowing and explaining the why and how of the procedures which are needed for a logical and correct solution of a mathematical problem is having or possessing a conceptual knowledge in mathematics.

As earlier noted, expertise in mathematics requires knowledge of both concepts and procedures. Knowledge of procedures is defined by Rittle-Johnson and Schneider (2012) as "knowing how or knowledge of the steps required to attain various goals", (p.3). They pointed out that this knowledge, unlike conceptual knowledge, is limited to a particular problem type. This means its development would be through constant practice of solving problems. Isleyen and Isik (2003) described procedural knowledge as knowing procedures, rules and algorithms required to solve mathematical problems. This description portrays this knowledge as mechanical, as it does not include conceptual understanding. This might be why Heibert and Lefevre (1986) asserted that procedural knowledge is "meaningful only if it is linked to a conceptual base", (p.). Khashan (2014), cited McGehee defining procedural knowledge as "the ability to explain or justify the way one resolves a given problem without knowing the reason behind applying a certain theory, process or law during problem solving process", (p. 182).

Procedural knowledge, in this study, can be said to be the ability to commit into memory the rules, procedures, principles and definitions of mathematics, and to recall them when solving problems without necessary having an understanding of them. That is if procedural knowledge is not linked to or does not have a base from conceptual knowledge, then such knowledge can be termed as 'mechanical'. This is so, as it will be difficult to apply such knowledge to unfamiliar situations or problems.

1.1 Relationship between conceptual and procedural knowledge in mathematics

Relationship between conceptual and procedural knowledge in mathematics has long been an issue of debate among researchers in mathematics education. Rittle-Johnson and Schneider (2012) pointed out that researchers in mathematics education have expressed four theoretical perspectives on the relationship between conceptual knowledge and procedural knowledge. The concept-first perspective (e.g. Halford, 1993; Gelman & Williams, 1998) assumes that children from the beginning gain conceptual knowledge, either through parents' explanations or by natural constraints, and then procedural knowledge is achieved through constant practice. On the other hand, the proponents of procedure-first perspectives (e.g Karmiloff-smith, 1992; Siegler & Stern, 1998) opine that children first learn procedural knowledge through inquisitiveness and later build conceptual knowledge about it. There is the inactivation view point (Resnick & Omanson, 1987, Resnick, 1982, Haaspasalo & Kadijevich, 2000) which says that conceptual knowledge and procedural knowledge develop independently. Even though Haaspasalo (2003) tried to identify certain relations between conceptual knowledge and procedural knowledge, he stressed that there is no general conclusion concerning relation between these two types of knowledge. The fourth view point is the iterative perspective (Rittle-Johnson & Siegler, 1998; Rittle-Johnson & Alibali, 1999; Baroody, 2003), which asserts that relation between conceptual knowledge and procedural knowledge is bidirectional. This means positive change in conceptual knowledge supports similar change in procedural knowledge and vice versa.

Zakaria, Yaakob, Maat and Adnan (2010) pointed out that knowing mathematics has to do with understanding of certain concepts and procedures. And therefore, there has to be link between conceptual and procedural knowledge. They said the mathematics curriculum should avail the students the opportunity to discover and make such links. This is imperative, as competence in mathematics is a combination of the knowledge of both concepts and procedures. Zakaria and Zaini (2009) investigated conceptual and procedural knowledge of trainee teachers in rational numbers. The study revealed that the prospective teachers' conceptual and procedural knowledge was above average. A study by Engelbrecht, Harding and Pothieter (2005) examined undergraduate students' performance and confidence in procedural and conceptual mathematics, and the results showed that performance of the students was better on conceptual knowledge than on procedural. Bryan (2002) investigating pre-service mathematics teachers to determine how well they know the subject they will teach, revealed that majority of the pre-service teachers were unable to justify their answers using conceptual knowledge. In fact, only about a quarter of the prospective teachers were successful in the use of conceptual knowledge to clarify their answers.

Stump (1996) carried out a study on secondary mathematics teachers to determine their knowledge on the concepts of slope. The study compared pre-service and in-service teachers' conceptual knowledge, and the results revealed that pre-service teachers had difficulty in differentiating between linear equations and others; they also were unable to answer questions which relate to rate of change. Similarly, Faulkenberry (2003) investigated secondary mathematics pre-service teachers conceptual knowledge of rational numbers, and the findings indicated that teachers who were inexperienced provided explanations that were based on knowledge of procedures. The study also revealed that experienced teachers' explanations were based on both knowledge of concepts and procedures. Similarly, Khashan (2014) conducted a study on elementary school teachers' conceptual and procedural knowledge of rational numbers. The study sample was 57 elementary mathematics teachers, consisting of 27 inexperienced and 30 experienced teachers. The study revealed that the teachers had average conceptual and procedural knowledge in rational numbers. It was also shown that significant difference existed between the teachers' conceptual and procedural knowledge, and the difference was in favor of procedural knowledge. In other words, the teachers performed better on procedural knowledge than on conceptual knowledge.

McGehee (1990) studied prospective secondary teachers' knowledge of the function concept and found that majority of the teachers have mastered procedural knowledge more clearly than their conceptual knowledge. The teachers' performance on problems that demanded procedural knowledge was better than on the ones that called for conceptual knowledge. Byrnes and Wasik (1991) conducted a study on the role of conceptual knowledge in mathematical procedural learning. They found that students gain conceptual understanding before procedural competence with fractions. Hence, they asserted that conceptual knowledge precedes procedural knowledge.

1.2 The importance of geometry

The knowledge of geometry is essential for understanding and appreciating the environment we live in (Zuya & Kwalat, 2015). Ozerem (2012) said, "studying geometry is an important component of learning mathematics because it allows students to analyze and interpret the world they live in as well as equip them with tools they can apply in other areas of mathematics" (p. 23). This assertion by Ozerem implies knowledge of geometry would afford the students the opportunity to be critical about their environment, and also to solve and appreciate real-life problems. According to NCTM (1989, 2000), knowledge of geometry benefits both teachers and students in other topics in the mathematics curriculum and other disciplines. Therefore, knowledge of geometry is an important component of mathematics knowledge, and hence necessary for competence in mathematics. Whether pre-service mathematics teachers possess these types of knowledge in geometry has not been documented in the literature. Hence, the need to investigate conceptual and procedural knowledge of pre-service mathematics teachers in geometry.

The knowledge of geometry is a combination of conceptual and procedural understanding. Conceptual knowledge in geometry can be said to be the understanding of the interrelationships among geometric concepts. In other words, it is the ability to correctly define and explain relationships among geometric concepts. On the other hand, procedural knowledge in geometry refers to recall of step by step procedures in solving a problem in geometry, and implementing these procedures to reach the solution. This study was targeted towards determining whether pre-service mathematics teachers possess both conceptual and procedural knowledge.

1.3 The Present Study

The present study focused on pre-service mathematics teachers' conceptual and procedural knowledge of geometry. Star (2016) asserted that most studies on conceptual and procedural knowledge have been on topics such as counting, single-digit and multi-digit addition and subtraction and also on fractions. Not much has been done regarding conceptual/procedural knowledge on topic areas such as geometry. In fact, Star (2016) said, "notably absent are the studies of the development of procedural and conceptual knowledge in algebra, geometry, and calculus" (p. 2). Many studies have been conducted on conceptual/procedural knowledge in fractions (e.g. Kerslake,1986; Hallett et al,2010; Hallett et al, 2012; Nune, Bryant, Baros & Sylva, 2012). Similarly, there are many studies of the development of conceptual and procedural knowledge in single-digit and multi-digit addition and subtraction (e.g. Canobi et al 2003, Mabbott & Bisant, 2003, Canobi, 2004, Gilmore & Bryant, 2006 & 2008).

1.4 Purpose of the Study

The main purpose of the study was to investigate conceptual and procedural knowledge of pre-service mathematics teachers in geometry. Specifically, the study aimed at:

1. Determining the extent to which pre-service mathematics teachers possess conceptual knowledge in geometry.
2. Determining the extent to which pre-service mathematics teachers possess procedural knowledge in geometry.
3. Determining whether pre-service mathematics teachers' conceptual knowledge of geometry is related to their procedural knowledge of geometry.

1.5 Research Questions

The following research questions were formulated to guide the study

1. To what extent do pre-service mathematics teachers possess conceptual knowledge in geometry?
2. To what extent do pre-service mathematics teachers possess procedural knowledge in geometry?
3. Is pre-service mathematics teachers' conceptual knowledge in geometry related to their procedural knowledge in geometry?

1.6 Hypothesis

H0: There is no significant relationship between the mean scores of pre-service mathematics teachers in conceptual and procedural knowledge of geometry

2.0 METHODOLOGY

The survey research design was implemented in this study. The independent variables of conceptual knowledge and procedural knowledge were addressed using quantitative methods.

The participants in this study were undergraduate students undergoing training to become secondary school mathematics teachers. They were 28 in number, and in their final year. They have had courses in both Mathematics and Education, and had also participated twice in the mandatory teaching practice exercise.

Two instruments were used to collect data for the study. Geometry Conceptual Knowledge Test (GCKT) and Geometry Procedural Knowledge Test (GPKT). Each instrument is a 15-item open-ended test. The items were given to three experts for face and content validity. The comments of the experts were considered in the final preparation of the test. The items were further pilot tested, and data obtained from the pilot testing were used for calculating the coefficients of reliability of the two tests. The correlation coefficients of Geometry conceptual knowledge test and Geometry procedural knowledge test were obtained as 0.76 and 0.81 respectively.

3.0 RESULTS

To answer research questions 1 and 2, the means and standard deviations of the pre-service mathematics teachers' scores in conceptual and procedural tests were calculated, and the results are shown in Table 1. From the table it can be seen that the respondents performed above average in conceptual knowledge test, but slightly below average in procedural knowledge test. Therefore, pre-service mathematics teachers in this study generally possess above average conceptual knowledge and below average procedural knowledge in geometry. The results also indicated wide gaps in individual performances, as can be seen from the standard deviations.

Table 1 Means and Standard deviation

	N	Mean	standard dev.
Conceptual knowledge	28	41.39	16.03
Procedural knowledge	28	22.75	15.72

To answer research question 3, Pearson product-moment correlation was run to determine the relationship between teacher's conceptual knowledge and procedural knowledge in geometry. There was a moderate, positive correlation between the two variables (r = .640, n = 28) which answers research question three. The summary of the calculation of the Pearson product-moment correlation is presented in Table 2. To further see the correlation, a scatter gram was plotted (concepts against procedures), and the diagram is as shown in Figure 1. It can be seen that a linear relationship exists between pre-service mathematics teachers' conceptual and procedural knowledge in geometry. Actually, Figure 1 shows that a line of best fit can be plotted, and it will have a positive gradient, indicating a positive and linear relation between the variables.

Table 2: Summary of Pearson correlation

		Concepts	Procedures
Concepts	Pearson Correlation	1	.640**
	Sig. (2-tailed)		0.000
	N	28	28
Procedures	Pearson Correlation	.640**	1
	Sig. (2-tailed)	0.000	
	N	28	28

**. Correlation is significant at the 0.05 level (2-tailed).

Only one hypothesis was tested in this study at alpha value of 0.05. And, as can be seen from Table 2, the correlation result clearly revealed that there is a statistically significant relationship between teachers' conceptual knowledge and procedural knowledge in geometry. This is because the p-value of 0.0001 is less than the alpha value of 0.05. Based on this, we can conclude that there is a significant relationship between the two variables. Hence there is sufficient evidence to reject the null hypothesis.

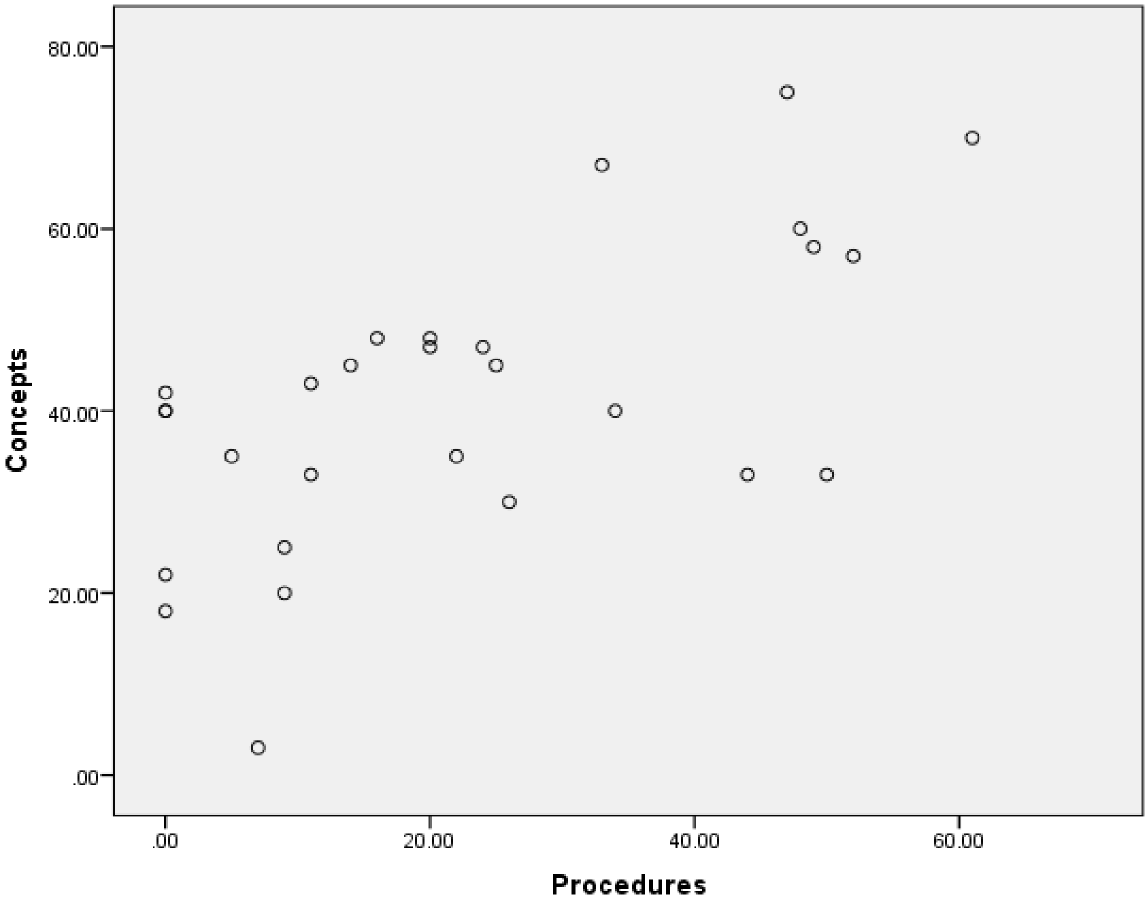

Figure 1: Positive linear relationship in geometry

4.0 DISCUSSION

One of the findings of the study is that pre-service mathematics teachers performed generally above average in conceptual knowledge test. Their performance on conceptual knowledge test was better than on procedural knowledge. This finding is similar to the finding of Zakaria and Zaini (2009), who reported that pre-service teachers' conceptual knowledge in rational number was above average. This is further corroborated by the findings of Khashan (2014), who also found that performance of elementary teachers was above average in rational numbers. In this study, performance of the respondents on conceptual knowledge was better than on procedural knowledge. Engelbrecht, Harding and Pothieter (2005) reported a similar finding; they said that undergraduate students' performance on conceptual knowledge was better than on procedural.

This study also revealed that pre-service mathematics teachers' conceptual knowledge test scores and procedural knowledge test scores correlated positively. This correlation was further tested and found that the relationship is statistically significant. This is in line with research findings documented in literature (e.g. Rittle-Johnson & Siegler, 1998; Rittle-Johnson & Alibali, 1999; Baroody, 2003), which indicated that positive relationship between conceptual and procedural knowledge exists.

5.0 CONCLUSION

Competence in mathematics requires knowledge of concepts and procedures. In this study, pre-service mathematics teachers were found to have scored above average in knowledge of concept test, but generally below average in knowledge of procedures. This probably means, though students can define certain concepts and rules in geometry, they are unable to transfer this knowledge into procedural skills to solve problems. This then suggests that teachers should stress knowledge of both concepts and procedures equally, as deficiency in one affects one's competence in the subject. The positive and significant relationship between the variables in this study does not mean equal knowledge of both concepts and procedures in geometry. This is evident from the performance of the pre-service mathematics teachers as shown in Table 1.

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