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A Family of Continuous Block A-Stable Third Derivative Linear Multistep Methods for Stiff Initial Value Problems

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ABSTRACT
In this paper, we present a family of block A-stable third derivative linear multistep methods of order p=k+2 for step numbers k=2,3,4 and 5 for the solution of stiff initial value problems. The newly constructed block methods are all A-stable, consistent, zero-stable and as such convergent. Numerical examples are considered to show the performance of the new methods.
Keywords: Continuous Block, A-stability, zero-stability, linear multistep methods, Third derivative

1. INTRODUCTION

Numerical solution of real life modelled problems in areas like engineering, science and social sciences often leads to stiff systems of first order Ordinary Differential Equations (ODEs) of the form,

$$y' = f(t, y), \quad y(t_0) = y_0 \tag{1}$$

The solution of (1) using the known analytical methods is not always easy and in some cases cannot even be solved at all using these methods. The Van Der pol's and the Robertson equations are good examples of (1) that cannot be solved analytically. In view of the importance of numerical methods in the solution of (1), Numerical analysts have developed methods for the numerical solution of both stiff and non-stiff problems of (1). Odekunle et al. [1] constructed a 4-point block method for direct integration of (1). In this paper, they proposed a new continuous linear multistep method, in which the approximate solution is the combination of power series and exponential function. Naghmeh et al. [2] developed new formulae of variable step 3-point block method for (1) to approximate three points concurrently at each iteration. Other authors who have done considerable work on the numerical solution of (1) for the first derivative case include Ibijola et al.[3], Ezzeddine and Hojjati [4], Chollom et al.[5], Kumleng et al. [6] and so on.

Second derivative methods have been studied also for the solution of (1) by many scholars like, Enright [7], Sahi et al.[8] and Kumleng et al.[9]. Ezzeddine and Hojjati [10] proposed third derivative method of order k + 3. In these methods the first, second and third derivatives of the solution were used to get high accuracy and an improved absolute stability region for the methods. Jator et al. [11] also constructed High-order continuous third derivative formulas with block extensions for the direct solution of the general second order ordinary differential equations. Recently, Akinfenwa et al. [12] constructed a family of continuous third derivative block methods for solving stiff systems of first order ordinary differential equations.

In what follows, we derive a family of continuous block A-stable third derivative linear multistep methods based on multistep collocation (see Lie and Norsett [13], Onumanyi et al. [14]). This family of block methods is self-starting and A-stable with good stability properties.

2 Derivation of the new block methods

In this section, a k-step third derivative method of the form

$$y_{n+k} - y_{n+k-1} = h\sum_{j=0}^{k} \beta_j(x) f_{n+j} + h^3 \gamma_k(x) q_{n+k} \quad (2)$$

is constructed for the solution (1) on the interval $[t_n, t_{n+k}]$ where $q = y''$, $\beta_j(x)$ and $\gamma_k(x)$ are continuous coefficients of the method.

Following Onumanyi et al. (1994), we obtained the continuous formulation of (2) for k=2 as

$$y(\varpi + x_n) = y_{n+1} + \left(\varpi - \frac{11}{12}\frac{\varpi^2}{h} + \frac{1}{3}\frac{\varpi^3}{h^2} - \frac{3}{8}h - \frac{1}{24}\frac{\varpi^4}{h^3}\right) f_n + \left(\frac{4}{3}\frac{\varpi^2}{h} - \frac{3}{4}h + \frac{1}{12}\frac{\varpi^4}{h^3} - \frac{2}{3}\frac{\varpi^3}{h^2}\right) f_{n+1}$$
$$+ \left(-\frac{5}{12}\frac{\varpi^2}{h} + \frac{1}{8}h - \frac{1}{24}\frac{\varpi^4}{h^3} + \frac{1}{3}\frac{\varpi^3}{h^2}\right) f_{n+2} + \left(-\frac{1}{24}h^3 + \frac{1}{6}\varpi^2 h - \frac{1}{6}\varpi^3 + \frac{1}{24}\frac{\varpi^4}{h}\right) q_{n+2} \quad (3)$$

where $\varpi = x - x_n$ and $\varpi \in [0, x_{n+2}]$

Evaluating (3) at $\varpi = 0$ and $2h$ yields the main and additional method (4) which are combined as block method to provide a global solution for (1).

$$y_{n+1} = y_n + \frac{h}{8}\left(3f_n + 6f_{n+1} - f_{n+2}\right) + \frac{h^3}{24} g_{n+2}$$
$$y_{n+2} = y_{n+1} - \frac{h}{24}\left(f_n - 14f_{n+1} - 11f_{n+2}\right) - \frac{h^3}{24} g_{n+2} \quad (4)$$

Similarly, the continuous formulations for k=3, 4 and 5 were obtained as

K=3

$$y(\varpi + x_n) = y_{n+2} + \left(\varpi - \frac{139}{132}\frac{\varpi^2}{h} + \frac{1}{2}\frac{\varpi^3}{h^2} - \frac{53}{165}h - \frac{29}{264}\frac{\varpi^4}{h^3} + \frac{1}{110}\frac{\varpi^5}{h^4}\right) f_n$$
$$+ \left(\frac{45}{22}\frac{\varpi^2}{h} - \frac{76}{55}h + \frac{35}{88}\frac{\varpi^4}{h^3} - \frac{3}{2}\frac{\varpi^3}{h^2} - \frac{2}{55}\frac{\varpi^5}{h^4}\right) f_{n+1}$$
$$+ \left(-\frac{3}{11}h - \frac{41}{88}\frac{\varpi^4}{h^3} - \frac{63}{44}\frac{\varpi^2}{h} + \frac{3}{2}\frac{\varpi^3}{h^2} + \frac{1}{22}\frac{\varpi^5}{h^4}\right) f_{n+2}$$
$$+ \left(-\frac{4}{165}h + \frac{29}{66}\frac{\varpi^2}{h} - \frac{1}{2}\frac{\varpi^3}{h^2} + \frac{47}{264}\frac{\varpi^4}{h^3} - \frac{1}{55}\frac{\varpi^5}{h^4}\right) f_{n+3}$$
$$+ \left(\frac{2}{165}h^3 - \frac{3}{22}\varpi^2 h + \frac{1}{6}\varpi^3 - \frac{3}{44}\frac{\varpi^4}{h} + \frac{1}{110}\frac{\varpi^5}{h^2}\right) q_{n+3}$$

K=4

$$y(\varpi + x_n) = y_{n+3} + \left(-\frac{691}{600}\frac{\varpi^2}{h} + \varpi + \frac{23}{36}\frac{\varpi^3}{h^2} - \frac{59}{320}\frac{\varpi^4}{h^3} - \frac{507}{1600}h + \frac{2}{75}\frac{\varpi^5}{h^4} - \frac{11}{7200}\frac{\varpi^6}{h^5}\right) f_n$$
$$+ \left(-\frac{69}{50}h + \frac{64}{25}\frac{\varpi^2}{h} - \frac{20}{9}\frac{\varpi^3}{h^2} + \frac{47}{60}\frac{\varpi^4}{h^3} - \frac{19}{150}\frac{\varpi^5}{h^4} + \frac{7}{900}\frac{\varpi^6}{h^5}\right) f_{n+1}$$
$$+ \left(\frac{6}{25}\frac{\varpi^5}{h^4} - \frac{213}{160}\frac{\varpi^4}{h^3} + \frac{19}{6}\frac{\varpi^3}{h^2} - \frac{549}{800}h - \frac{66}{25}\frac{\varpi^2}{h} - \frac{19}{1200}\frac{\varpi^6}{h^5}\right) f_{n+2}$$

$$+\left(-\frac{18}{25}h-\frac{20}{9}\frac{\varpi^3}{h^2}-\frac{31}{150}\frac{\varpi^5}{h^4}+\frac{128}{75}\frac{\varpi^2}{h}+\frac{21}{20}\frac{\varpi^4}{h^3}+\frac{13}{900}\frac{\varpi^6}{h^5}\right)f_{n+3}$$
$$+\left(\frac{1}{15}\frac{\varpi^5}{h^4}+\frac{23}{36}\frac{\varpi^3}{h^2}-\frac{7}{1440}\frac{\varpi^6}{h^5}-\frac{19}{40}\frac{\varpi^2}{h}-\frac{61}{192}\frac{\varpi^4}{h^3}+\frac{33}{320}h\right)f_{n+4}$$
$$+\left(-\frac{9}{400}h^3+\frac{3}{25}\varpi^2 h-\frac{1}{6}\varpi^3+\frac{7}{80}\frac{\varpi^4}{h}-\frac{1}{50}\frac{\varpi^5}{h^2}+\frac{1}{600}\frac{\varpi^6}{h^3}\right)q_{n+4}$$

K=5

$$y(\varpi+x_n)=y_{n+4}+\left(-\frac{4342\,h}{14385}-\frac{20269\varpi^2}{16440\,h}+\frac{55\varpi^3}{72h^2}-\frac{1727}{6576}\frac{\varpi^4}{h^3}+\frac{209}{4110}\frac{\varpi^5}{h^4}-\frac{32}{6165}\frac{\varpi^6}{h^5}+\varpi+\frac{5}{23016}\frac{\varpi^7}{h^6}\right)f_n$$
$$+\left(\frac{1274}{2055}\frac{\varpi^5}{h^4}-\frac{1075}{274}\frac{\varpi^2}{h}-\frac{8429}{3288}\frac{\varpi^4}{h^3}+\frac{13}{3836}\frac{\varpi^7}{h^6}+\frac{185}{36}\frac{\varpi^3}{h^2}-\frac{1864}{4795}h-\frac{361}{4932}\frac{\varpi^6}{h^5}\right)f_{n+2}$$
$$+\left(-\frac{12383}{16440}\frac{\varpi^5}{h^4}+\frac{1879}{19728}\frac{\varpi^6}{h^5}+\frac{2975}{822}\frac{\varpi^2}{h}-\frac{107}{23016}\frac{\varpi^7}{h^6}-\frac{185}{36}\frac{\varpi^3}{h^2}-\frac{23312}{14385}h+\frac{37501}{13152}\frac{\varpi^4}{h^3}\right)f_{n+3}$$
$$+\left(\frac{671}{1370}\frac{\varpi^5}{h^4}+\frac{11}{3288}\frac{\varpi^7}{h^6}-\frac{11471}{6576}\frac{\varpi^4}{h^3}-\frac{2225}{1096}\frac{\varpi^2}{h}-\frac{323}{4932}\frac{\varpi^6}{h^5}-\frac{346}{2055}h+\frac{215}{72}\frac{\varpi^3}{h^2}\right)f_{n+4}$$
$$+\left(-\frac{40}{959}h-\frac{55}{72}\frac{\varpi^3}{h^2}+\frac{3649}{197280}\frac{\varpi^6}{h^5}+\frac{12043}{26304}\frac{\varpi^4}{h^3}-\frac{4373}{32880}\frac{\varpi^5}{h^4}+\frac{1399}{2740}\frac{\varpi^2}{h}-\frac{15}{15344}\frac{\varpi^7}{h^6}\right)f_{n+5}$$
$$+\left(\frac{32}{2877}h^3-\frac{15}{137}\varpi^2 h+\frac{1}{6}\varpi^3-\frac{225}{2192}\frac{\varpi^4}{h}-\frac{5}{1096}\frac{\varpi^6}{h^3}+\frac{1}{3836}\frac{\varpi^7}{h^4}+\frac{17}{548}\frac{\varpi^5}{h^2}\right)q_{n+5}$$

These continuous formulations are evaluated at $\varpi=\{0,h,3h\}$, $\varpi=\{0,h,2h,4h\}$ and $\varpi=\{0,h,2h,3h,5h\}$ for k=3,4 and 5 respectively to yield the following block methods

$$\begin{aligned}
y_{n+1}&=y_{n+2}+\frac{h}{40}\left(f_n-19f_{n+1}-25f_{n+2}+3f_{n+3}\right)-\frac{h^3}{60}g_{n+3}\\
y_{n+2}&=y_n+\frac{h}{165}\left(53f_n+228f_{n+1}+45f_{n+2}+4f_{n+3}\right)-\frac{2h^3}{165}g_{n+3}\\
y_{n+3}&=y_{n+2}+\frac{h}{1320}\left(17f_n-123f_{n+1}+855f_{n+2}+571f_{n+3}\right)-\frac{19h^3}{660}g_{n+3}
\end{aligned}\tag{5}$$

$$\begin{aligned}
y_{n+1}&=y_{n+3}+\frac{h}{90}\left(f_n-34f_{n+1}-114f_{n+2}-34f_{n+3}+f_{n+4}\right)\\
y_{n+2}&=y_{n+3}-\frac{h}{1600}\left(11f_n-96f_{n+1}+874f_{n+2}+896f_{n+3}-85f_{n+4}\right)-\frac{11}{1200}h^3g_{n+4}\\
y_{n+3}&=y_n+\frac{h}{1600}\left(507f_n+2208f_{n+1}+1098f_{n+2}+1152f_{n+3}-165f_{n+4}\right)+\frac{9}{400}h^3g_{n+4}\\
y_{n+4}&=y_{n+3}-\frac{h}{14400}\left(83f_n-608f_{n+1}+2202f_{n+2}-10112f_{n+3}-5965f_{n+4}\right)-\frac{9}{400}h^3g_{n+4}
\end{aligned}\tag{6}$$

$$\begin{aligned}
y_{n+1}&=y_{n+4}+\frac{h}{306880}\left(3804f_n-120447f_{n+1}-364152f_{n+2}-292002f_{n+3}-162372f_{n+4}+14529f_{n+5}\right)-\frac{117h^3}{15344}q_{n+5}\\
y_{n+2}&=y_{n+4}-\frac{h}{172620}\left(250f_n-3443f_{n+1}+69112f_{n+2}+213302f_{n+3}+69062f_{n+4}-3043f_{n+5}\right)-\frac{5h^3}{2877}q_{n+5}\\
y_{n+3}&=y_{n+4}+\frac{h}{920640}\left(2516f_n-21677f_{n+1}+94488f_{n+2}-556742f_{n+3}-476812f_{n+4}+37587f_{n+5}\right)-\frac{271h^3}{46032}q_{n+5}\\
y_{n+4}&=y_n+\frac{h}{14385}\left(4342f_n+21272f_{n+1}+5592f_{n+2}+23312f_{n+3}+2422f_{n+4}+600f_{n+5}\right)-\frac{32h^3}{2877}q_{n+5}\\
y_{n+5}&=y_{n+4}+\frac{h}{2761920}\left(8636f_n-68599f_{n+1}+251336f_{n+2}-607154f_{n+3}+2072476f_{n+4}+1105225f_{n+5}\right)-\frac{863\,h^3}{46032}q_{n+5}
\end{aligned}\tag{7}$$

3. Analysis of the Method

We present here the analysis of the block methods in (4),(5),(6) and (7). Convergence which is an important property required of all good linear multistep methods shall be investigated for the block methods and the regions of absolute stability plotted.

3.1 Local Truncation Error

In the spirit of Fatunla [15] and Lambert [16], the local truncation error associated with the block methods is the linear difference operator

$$L[Y(x):h]=\sum_{j=0}^{k}\left\{\alpha_j Y(x+jh)-hY'\beta_j(x+jh)-h^3Y'''\gamma_j(x+jh)\right\} \qquad (8)$$

We assume that $Y(x)$ is sufficiently differentiable, and so the terms of (11) can be expanded as Taylor series about x to give the expression

$$L[Y(x):h]=C_0Z(x)+C_1hZ'(x)+...+C_qh^qZ^{(q)}(x)+... \qquad (9)$$

where

$$C_0=\sum_{j=0}^{k}\alpha_j$$

$$C_1=\sum_{j=1}^{k}j\alpha_j-\sum_{j=0}^{k}\beta_j$$

$$C_2=\tfrac{1}{2}\sum_{j=1}^{k}j^2\alpha_j-\sum_{j=1}^{k}j\beta_j-\sum_{j=0}^{k}\gamma_j$$

$$\vdots$$

$$C_q=\tfrac{1}{q!}\sum_{j=1}^{k}j^q\alpha_j-\tfrac{1}{(q-1)!}\sum_{j=1}^{k}j^{q-1}\beta_j-\tfrac{1}{(q-3)!}\sum_{j=1}^{k}j^{q-3}\gamma_j,\ q=3,4,..$$

A block method is said to be of order p if $\bar{C}_0=\bar{C}_1=...=\bar{C}_p=\bar{0},\quad \bar{C}_{p+1}\neq\bar{0}.$

$\bar{C}_{p+1}$ is called the error constant and the local truncation of the method is given as

$$\bar{t}_{n+k}=\bar{C}_{p+1}h^{(p+1)}y^{(p+1)}x_n+O(h^{(p+1)}). \qquad (10)$$

The block method (4) is of order $p=(4,4)^T$ and has an error constant of $\left[\frac{11}{480},\frac{17}{1440}\right]^T$

The block method (5) is of order $p=(5,5,5)^T$ with an error constant of $\left[\frac{1}{160},\frac{1}{99},\frac{83}{15840}\right]^T$

The block method (6) is of order $p=(6,6,6,6)^T$ and has an error constant of . $\left[\frac{1}{756},\frac{629}{252000},\frac{317}{28000},\frac{2159}{756000}\right]^T$. The block method (7) is of order $p=(7,7,7,7,7)^T$ and with an error constant of $\left[\frac{299}{122752},\frac{11}{103572},\frac{449}{368256},\frac{16}{2055},\frac{29101}{16571520}\right]^T$.

Since the block methods are all of order $p\geq 1$, they are all consistent. Henrici (17)

3.2 Zero Stability of the Block method

Definition 3.2.1

Any of the block methods (4) –(7) is said to be zero-stable provided the roots $R_j, j=1,...,k$ of the first characteristic polynomial $\rho(R)$ specified by

$$\rho(R) = \det\left[\sum_{i=0}^{k} A^{(i)} R^{k-i}\right] = 0 \qquad (11)$$

satisfies $\left|R_j\right| \leq 1, j = 1,..., k.$ and for those roots with $\left|R_j\right| = 1,$ the multiplicity does not exceed 1.
Applying the usual test equations

$$y' = \lambda y\,, \quad y''' = \lambda^3 y$$

to the block method (4) with $z = \lambda y$ and solving the characteristic equation

$$\det\left(r(A - Cz - Dz^3) - B\right) = 0 \qquad (12)$$

$$A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} \frac{3}{4} & \frac{1}{8} \\ \frac{7}{12} & \frac{11}{24} \end{pmatrix}, \quad D = \begin{pmatrix} 0 & \frac{1}{24} \\ 0 & -\frac{1}{24} \end{pmatrix}$$

for r at $z = 0$ yields the following roots {0,1}. The block method is therefore zero stable by the above definition. Similarly, all the other methods were found to be zero-stable.

3.3 Convergence

The block methods are all convergent since they are all consistent and zero stable. (See Henrici [17])

3.4 Region of Absolute Stability of new method

From (12), we obtain the stability function R(z) which is a rational function of real coefficients. We obtain as follows the R(z) for k=2,3,4 and 5 as

K=2

$$R(z) = -\frac{3\,(7z + 12)}{2z^4 - 15z^2 + 39z - 36}$$

K=3

$$R(z) = \frac{1}{2}\,\frac{161z^2 + 453z + 440}{9z^5 - 78z^3 + 247z^2 - 360z + 220}$$

K=4

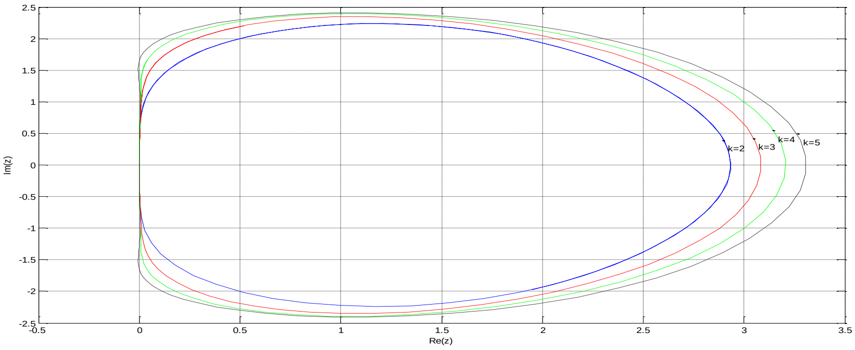
$$R(z) = -\frac{1149z^3 + 4066z^2 + 6820z + 4500}{144z^6 - 1390z^4 + 5055z^3 - 9450z^2 + 9780z - 4500}$$

K=5

$$R(z) = \frac{1}{2}\,\frac{131478z^4 + 535525z^3 + 1188025z^2 + 1377025z + 690480}{9000z^7 - 94660z^5 + 381489z^4 - 841800z^3 + 1153200z^2 - 932400z + 345240}$$

respectively.

The boundary locus plots of the stability domain for the block methods are given in figure 1. Hence we conclude that all the block methods are A-



stable.

Fig 1: Stability Regions of the Block methods for k=2,3,4,5

4 Numerical experiments

In this section we shall test the new A-stable block methods for k=4 and 5 in order to ascertain their suitability for solving (1)

Problem 1. Consider the stiff system:

$$y_1' = -y_1 - 15y_2 + 15e^{-x}$$
$$y_2' = 15y_1 - y_2 - 15e^{-x}$$
$$y_1(0) = y_2(0) = 1,\ h = 0.01,\ \ y_1(x) = y_2(x) = e^{-x}, 0 \le x \le 10$$

This system has eigenvalues of large modulus lying close to the imaginary axis$-1 \pm 15i$

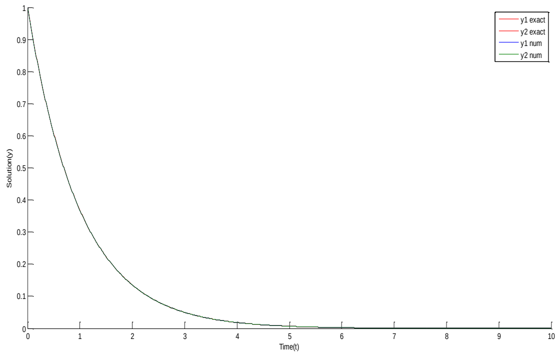


Fig 2: Solution curve for problem1 using the block method in (6)

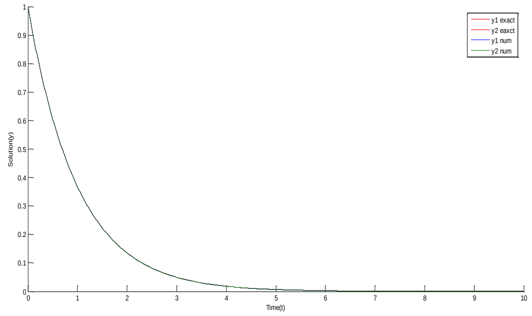


Fig 3: Solution curve for problem1 using the block method in (7)

Problem 2.

$$\begin{bmatrix} y_1'(x) \\ y_2'(x) \end{bmatrix} = \begin{bmatrix} 998 & 1998 \\ -999 & -1999 \end{bmatrix} \begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix}, \quad \begin{bmatrix} y_1(0) \\ y_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and the exact solution is given by the sum of two decaying exponential components,

$$\begin{cases} y_1(x) = 4e^{-x} - 3e^{-1000x} \\ y_2(x) = -2e^{-x} + 3e^{-1000x} \end{cases}$$

The stiffness ratio is 1:1000. We solve the problem in the interval [0, 10] and the computed results are as shown in Fig 4 and Fig 5 .

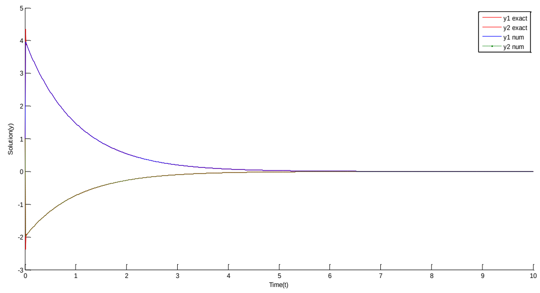


Fig 4: Solution curve for problem 2 using the block method in (6)

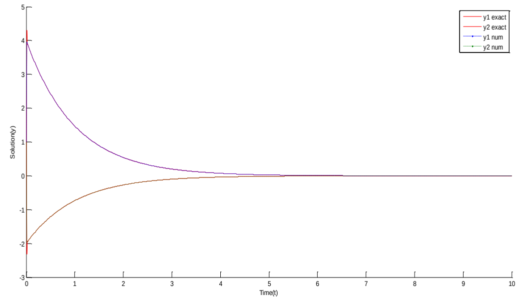


Fig 5 : Solution curve for problem2 using the block method in (7)

Problem 3.

We consider the linear system with constant coefficients given as

$$y' = \begin{pmatrix} -20 & -0.25 & -19.75 \\ 20 & -20.25 & 0.25 \\ 20 & -19.75 & -0.25 \end{pmatrix} y, \ \ y(0) = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \quad 0 \le x \le 10, \ h = 0.01$$

The three components of the theoretical solution of the problem are given as

$$y(x) = \tfrac{1}{2} \begin{pmatrix} e^{-0.5x} + e^{-20x}(\cos(20x) + \sin(20x)) \\ e^{-0.5x} - e^{-20x}(\cos(20x) - \sin(20x)) \\ -e^{-0.5x} - e^{-20x}(\cos(20x) - \sin(20x)) \end{pmatrix}$$

Table 1: Absolute Errors for problem 3 using the Block method in (6)

x	y1	y2	y3
0.01	6.0210302177E-07	3.8698563148E-07	3.8698563154E-07
1.00	8.7707618945E-15	1.7991164114E-13	1.8002266344E-13
2.00	1.1102230246E-16	1.1102230246E-16	8.3266726847E-17
3.00	8.3266726847E-17	9.7144514655E-17	8.3266726847E-17
4.00	2.7755575616E-17	2.7755575616E-17	2.7755575616E-17
5.00	2.0816681712E-17	2.0816681712E-17	2.0816681712E-17
6.00	2.7755575616E-17	2.4286128664E-17	2.7755575616E-17
7.00	2.0816681712E-17	2.2551405188E-17	2.4286128664E-17
8.00	1.2143064332E-17	1.2143064332E-17	1.2143064332E-17
9.00	1.0408340856E-17	9.5409791179E-18	1.0408340856E-17
10.00	8.6736173799E-18	8.6736173799E-18	8.2399365109E-18

Table 2: Absolute Errors for problem 3 using the Block method in (7)

x	y1	y2	y3
0.01	1.4644512591E-07	1.6641100192E-08	1.6641100720E-08
1.00	2.4702462298E-14	2.7478019859E-14	2.7700064464E-14
2.00	2.2204460493E-16	2.4980018054E-16	2.2204460493E-16
3.00	2.3592239273E-16	2.4980018054E-16	2.4980018054E-16
4.00	2.7755575616E-16	2.6367796835E-16	2.7755575616E-16
5.00	2.0122792321E-16	2.0816681712E-16	2.0122792321E-16
6.00	1.5265566589E-16	1.5265566589E-16	1.5265566589E-16
7.00	9.0205620751E-17	9.0205620751E-17	9.1940344227E-17
8.00	6.4184768611E-17	6.4184768611E-17	6.4184768611E-17
9.00	3.9898639947E-17	3.9898639947E-17	3.9898639947E-17
10.00	2.5587171271E-17	2.5587171271E-17	2.6020852140E-17

CONCLUSION

A new family of A-stable block methods that are self-starting have been introduced for the solution of stiff systems. The analysis of the stability properties shows that the methods are all A-stable and convergent. The numerical experiments considered showed that the methods compete favourably with existing ones.

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