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Analysing the Effect of Length on Pressure Drawdown in a Horizontal Well

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ABSTRACT
Horizontal drilling is an advanced and modem technology for making holes. It is being increasingly used for oil field development to overcome some technical problems such as water/gas conning and cusping. It is also used for low-cost, high efficiency development of small oil fields, and its use will continue well into the next millennium. The performance of oil producing reservoirs is largely determined by the nature of the natural energy available for moving the oil to the wellbore and the manner in which it is actually used during production. Optimizing the exploitation of a reservoir calls for the greatest production of oil at the least possible expenditure of reservoir energy. In a horizontal well, the length of the perforated horizontal drain section is a critical factor determining the pressure drawdown in the reservoir, and hence its performance. Moreover, the risk of missing the target increases as the length of the well increases. Hence, care must be taken in determining the optimum length of the horizontal drainhole to achieve a balance between the cost of drilling, completion and the attendant operational risks against the well productivity. In this study, the pressure drop in an oil reservoir penetrated by a horizontal well as a function of the length of the perforated horizontal section is investigated mathematically. The resulting 3-D non-linear differential equation for an infinite slab reservoir model was solved for the pressure drawdown function by the Lord Kelvin’s point source solution method. A numerical method approach was used to obtain the actual pressure drawdown from the pressure drawdown function. The study shows that the pressure drawdown decreases as the length of the perforated horizontal section increases. However, at late times, when pseudo-radial flow has been achieved, the reduction in pressure drawdown with increase in the length of the drainhole becomes negligible.
Keywords: Horizontal well, perforation, well productivity, drainhole, pressure, drawdown, wellbore , production and pressure buildup and pressure drawdown

INTRODUCTION

Since horizontal wells are increasingly being used, a definite need exists for information on how the length of the horizontal drain section affects pressure drawdown in the reservoir. This is to ensure optimal utilization of reservoir energy to the end that ultimate recovery may be increased.

In recent years, several papers have appeared in the literature on the subject of pressure transient behaviour of horizontal wells for a variety of reservoir and boundary conditions. These Works are all tailored towards the analysis of pressure buildup, pressure drawdown, and productivity evaluation, The mathematical development of most of the solutions were obtained by solving the 3-D diffusivity equation using the instantaneous source and Greens function method or Gringarten and Rameyu. The resulting pressure drop functions were solved with either Laplace transforms. They arrived at very complex analytic solutions for pressure drawdown and buildup. The results also did not show explicitly how the pressure drawdown is affected by the length of the horizontal well.

Kuchuk et al reported a theoretical equation to calculate the pressure drawdown in a 3-0 reservoir bounded in two dimensions. They concluded that the image sum representation could be used instead of the Fourier series to model the pressure response of the horizontal well bounded by the upper and lower no-flow boundaries in the vertical- direction. However, the report did not include the actual application of either methods or their uses to investigate the effect of length on pressure drawdown in the reservoir.

Another useful method which has hardly been employed due to its complexity, is the Lord Kelvin's instantaneous point source solution. The method was used by Nisle[12] in studying the effect of partial penetration on pressure buildup in oil wells. Cinco et al[13] also applied the Lord Kelvin s instantaneous point source method as suggested by Nisle', to study the unsteady state pressure distribution created by a directionally drilled well. There is a noticeable absence in the petroleum literature of studies dealing with the effect of horizontal well length on pressure drawdown using the Lord Kelvin's instantaneous point source solution.

This study reports the derivation of a numerical relationship between the pressure drawdown in a reservoir and the length of the horizontal drain section of a horizontal well in an under-saturated oil reservoir.

Physical and Mathematical Models

The physical model considered in this study consists of a line - source well located in a slab homogeneous medium of uniform thickness. This is possible where a reservoir is close to being continuous shale. A schematic diagram of the infinite slab reservoir is presented in fig 1.

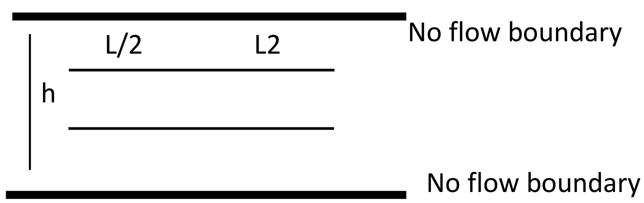

Fig. 1. Horizontal Well in an Infinite Slab Reservior

A horizontal well, with the length of the horizontal drain Section equal to L. and radius r penetrates the laterally infinite reservoir of thickness h. The well is assumed to be parallel to the top and bottom boundaries of the slab reservoir and is located at an elevation, z. it is assumed to produce at a constant rate, q. The flow of a slightly compressible fluid of constant compressibility and viscosity is assumed throughout the medium. Gravity effects are assumed to be negligible. Production is from a single well centered in the reservoir In actual fact most horizontal wells are crooked and are not parallel to the formation bedding planes Permeability porosity and viscosity are assumed independent of pressure and time, that is, the formation fluid properties are independent of pressure along the wellbore is assumed uniform, that is, infinite and only the production from the horizontal section is used to evaluate the performance of the well. It is assumed that skin and wellbore storage effects are negligible.

Fluid flow in a reservoir with horizontal well is a three - dimensional problem with anisotropy playing a prominent role in the performance of the well[2] Let x- axis be in the same direction of the horizontal wellbore , y-axis be perpendicular to x-axis in horizontal plane, and z-axis be the vertical axis The pressure behaviour during flow in the medium is given by the diffusivity equation:

$$\frac{\delta^2 P}{\delta x^2}+\frac{\delta^2 P}{\delta y^2}+\frac{\delta^2 P}{\delta z^2}=\frac{\mu c\phi}{k_y}\frac{\delta P}{\delta t}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where x, y, z are now the principal axes of permeability. The pressure in the drainage volume before the well is produced at t =0 is uniform and equal to Pi. We start to withdraw a quantity of fluid at the rate, q, from the well. We want to find the pressure drop, P (Pi - P), as a function of time and space, caused by unit length of the horizontal well. Thus the initial and boundary conditions of the model expressed mathematically are:

$$\frac{\delta P}{\delta z} = 0 \; at \; z = 0 \; and \; z = h \ldots\ldots\ldots\ldots(2a)$$

$$P = P_I \; at \; x_i \; y = \infty \quad \ldots\ldots\ldots\ldots\ldots\ldots(2b)$$

$$P = P_I \; at \; t = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2c)$$

Solution Of The Basic Differential Equation (Eq.1)

The pressure drop at a point, created by an instantaneous horizontal line source of finite length, L, in an infinite slab porous medium of thickness h. is given by;

$$\_P_1(x,y,z,z_w,L,h,t) = \frac{q\exp}{4\phi c(\pi n_h t)}(-(y-y^{/})^2/4n_y t)$$

$$x\frac{1}{2}\{erf\frac{(x-L/2)}{\sqrt{(4n_x t)}} + \frac{erf(x+L/2\}}{\sqrt{(4n_x t)}}$$

$$x\sum_{n=-x}^{x}(\exp(-(z-z_w+2nh)^2/4n_z t) + \exp(-(z+z_w+2nh)^2/4n_z t))\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3a)$$

Fig. 2 shows the images of the horizontal well in an infinite slab reservoir.

In terms of Fourier functions, eq.3a can be expressed as:

$$\_P_2(x,y,z,z_w,L,h,t) = \frac{q\exp}{4\phi c(\pi n_h t)}(-(y-y^{/}) \;\; /4n_y t)$$

$$x\frac{1}{2}\{erf((x-L/2)/\sqrt{(4}n_x t)) + erf((x+L/2)/\sqrt{4}n_x t))\}$$

$$x2\sqrt{(\pi}n_z t)/h(1+2\sum_{n=1}^{x}(\cos n\pi z/h)\cos(n\pi z_w/h)\exp(-n_z n^2\pi^2 t/h^2))\ldots\ldots\ldots\ldots\ldots\ldots(3b)$$

Integration of eq.3b with respect to time gives the pressure drop created by a continuous horizontal line source:

$$\_P_2(x,y,z,z_w,L,h,t) = \frac{q\mu B}{4\phi c(\pi n_h)h}\sqrt{(\pi k_z)}_0^T \_\{\exp(-(y-y^{/})^2/4n_y t\}$$

$$x\frac{1}{2}\{erf\frac{(x-L/2)}{\sqrt{(4n_x t)}}+erf\frac{(x+L/2)}{\sqrt{(4n_x t)}}\}$$

$$x\{1+2\sum_{n=1}^{x}\cos(n\pi z/h)\cos(n\pi z_w/h)\exp(-n_z n^2\pi^2 t))dt/\sqrt{t}\}.........................(4)$$

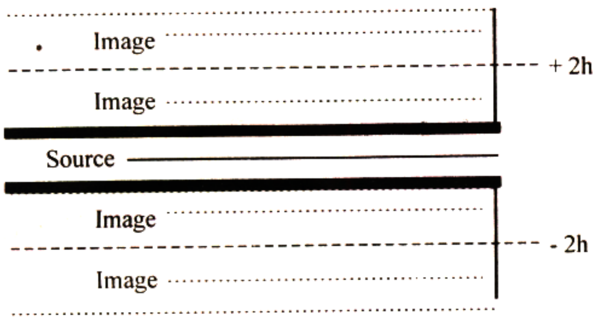

Fig. 2: Horizontal Well in an Infinite Slab Reservoir and Multiple Images

Eq.4 is the formal representation of the integration that must be performed in order to obtain the pressure drop due to a continuous source of length, L. in an infinite slab of thickness, h. It is apparent that the analytical solution of eq.4 presents certain difficulties - In order to examine the effect of length on pressure drawdown, therefore, numerical methods will be necessary.

For convenience we express the variables in their dimensionless forms. Substituting dimensionless parameters, we have:

$P_D(x_D, y_D, z_D, z_{WD}, L_D, t_D)$ =

$$\sqrt{(\pi/4)}\sqrt{(kh/ky)}I_0^t(\exp(-y^2{}_D/4t_D)\{erf\frac{(\sqrt{(k_h/k_x)}-x_D)}{2\sqrt{t_D}}+\frac{(\sqrt{(k_h/k_x)}-x_D)}{2\sqrt{t_D}}\}$$

$$x\{1+2\sum_{n=1}^{\infty}\exp(-n_2\pi^2 L_D{}^2 t_D)\cos(n\pi z_D)\cos(n\pi z_{WD}))dt_D/\sqrt{(t_D)}\}...............(5)$$

In eq. 5, one needs only account for a finite number of images, n, since the distant images would have negligible effect. The more distant the image is, the longer it will take its effect to be felt at the producing well.

Application and Results

It is not apparent from an examination of eq.5 how the length of a horizontal well affects pressure drawdown in the reservoir. In order to investigate this question, numerical calculations must be employed over a suitable range of values of the parameters.

The model (eq. 5) was tested and validated with an example taken from the literature. The example is that of a test in a homogeneous laterally infinite reservoir. The drilled horizontal welt length is 1,150ft but a production logging survey shows producing length (perforated length) of about 900ft. The well and reservoir parameters are shown in Table 1.

To obtain the wellbore pressure response, PwD, the PD function (eq. 5), was evaluated at approximate XD, yD and ZD (ZD = ZWD + rWD,

yD = rwD, XD = 0.708, 0.701, 0.698, 0.688, 0.601) for different values of L, including the actual producing length.

PwD is the unit response function of the full system that is the dimensionless wellbore pressure including the effect of wellbore storage for a constant flow rate at the measuring point and the skin. This is different from the unit response function, P_D. of the formation alone.

Solution For Infinite Conductivity Well

If eq.5 is evaluated at x =+- L/2. it gives the uniform flux solution for a horizontal well of length producing either from the right or left end of the wellbore. The conductivity of real wells, however, would be close to infinity, and hence the pressure distribution, instead of the flux distribution, would be uniform along the well (infinite conductivity well) Ref. 6 indicates that value for the infinite conductivity horizontal well solution should depend on LD.

The conventional method that can be used to approximate the infinite conductivity solution with the uniform flux solution is by using an equivalent pressure point, that is, to measure the pressure at XD = XD, where X_D is some suitably determined value. This approach was first suggested by Muskat who suggested the value of X_D =O.75 from any extremity of the well. It was studied in more detail by Gringarten and Ramey[14], who gave values Of X_D for a well in infinite isotropic medium for various values of r_w/L. The values of x*D, used in this study were obtained from the result of ref. 14 for corresponding values of r_{weq}/L.

In general, the location of the equivalent pressure point will vary with time Such variations are neglected in this study. It is assumed that the equivalent wellbore point computed from the stabilized pressure distribution (late – time solution) can be used at all times because the eary-time flux distribution is nearly uniform and is fixed at late times. Thus, the exact location of the equivalent point should have a small effect on the accuracy of pressure values during the transition period between the early-time uniform flux and the late-time stabilized conditions.

The equivalent well radius, r_{weq}, may be computed by:

$$r_{weq} = 0.5 r_w ((k / k_Z)^{0.25} + (k_Z / k)^{0.25}) \text{..........}(6)$$

In the dimensionless isotropic coordinate system, the equivalent wellbore radius is:

$$r_{wD} = (r_{weq} / 2L_w)(\sqrt{(}k_z / k_y) + \sqrt{(}k_x / k_y)) \text{....................}(7)$$

with the line source approximation for a horizontal well located at the reservoir mid-height. The method of least squares estimation is used for the solution of eq.5 because many parameters are involved. For the purpose of the least squares approximation, we set:

$$GFI = \exp(-y^2{}_D / 4t_D)\{erf\,\frac{(\sqrt{(}K_h / K_x) - X_D)}{2\sqrt{t_D}} + erf\,\frac{(\sqrt{(}K_h / K_x) + X_D)}{2\sqrt{t_D}}\}$$

$$x\{1 + 2\sum_{n-1}^{x} \exp(-n_2 \pi^2 L_D{}^2 t_D) \cos(n\pi z_D) \cos(n\pi z_{WD}))dt_D / \sqrt{(}t_D)\} \text{........................}(8)$$

With the foregoing in view, a software computer programme was developed for the solution of eq5. Fig.3 shows the flow chart for the computer programme.

The programme was used to generate values of GFI for different values of tD (Table 3). The integral in eq.5 was then calculated by plotting (GIF versus t_D, and finding the area under the curve from 0 to tD.

Short-Time Pressure Behaviour

The process of generating values of GFI using the developed computer programme for different values of t_D runs Into difficulty for early times ($t_D<10^{-1}$). The programme gives overflow problem. However, this problem was solved as discussed by Ozkan et al[8], Nisle[12], and Gringarten and Ramey[14].

At early times (tD< 10^{-1}), the horizontal well behaves as a vertical well producing an infinite reservoir of height L, and the value of the integral (eq.5) approaches the Ei-function solution.

The term $\{erf \frac{(\sqrt{(K_h / K_x) - X_D)}}{2\sqrt{t_D}} + \frac{erf(\sqrt{(K_h / K_x) + X_D)}}{2\sqrt{t_D}}\}$ in eq.5 is also constant at early times.

Therefore, the asymptotic form of eq.5 in the image sum representation during this early period is:

$$P_D(/x_D \le 1/, y_D, z_D, z_{WD}, L_D, t_D) = -(1/4\alpha L_D)Ei(-((z_D - z_{WD}/L_D)^2 + y_D{}^2)/4t_D) \text{..........}(9)$$

where $\alpha = 1$for$/x_D/\le 1$ and 2 for $/x_D/\le 1$. For $/x_D/\le 1$, P_D=0 no flow exists beyond the tips of the well and fluid flows vertically into the horizontal well. In the case considered $\alpha = 1$since $/x_D/\le 1$.

For these values of t_D, published Ei-function solution tables were used to evaluate P_{WD} at the corresponding t_D. For large values of t_D, the integral approaches zero. This can be observed from Table 2. Moreover, the well bore pressure response approach the same value.

The approximate expression for the duration of the early time period is:

$$(\sigma_D{}^2 / 20)$$

$$t_D \le \min\{(z_D + z_{WD})^2 / (20L_D{}^2)$$

$$(z_D + z_{WD}) - 2)^2 / (20L_D{}^2) \text{...............}(10)$$

where $\sigma_D = 1 - /x_D/$and 2 for $/x_D/\le 1$ and $/x_D/ = 1$ respectively.

To evaluate the wellbore pressure, only the horizontal section, excluding the 50 ft of vertical penetration, was considered.

Validation Of Result

The solution of eq.5 as derived in this study yields the dimensionless sand-face pressure response, P_{WD} at corresponding dimensionless time, t_D. To compute the actual pressure response, P(t) for a real time, tD, the relation between the actual dimensionless quantities, eq.1-6 and eq1-7 were applied, giving

$$t_D = 474.700t / L^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

$$\_\, P = 865.196 P_D \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Eq. 11 and eq.12 were then used to obtain P for L =9O0 ft, the productive length of the well. The results are resented in Table 5. The results obtained for the pressure drawdown for L= 90O ft by USing the model developed here are in good agreement with the results of the model applied in the literature also presented in Table 5.

DISCUSSION OF RESULT

The pressure drawdown created by the infinite conductivity line source horizontal wells were computed for several values of length of horizontal drain section and flow rates. The results for flow rate q =500 bbl/d are shown in fIg.3. It can be observed from the figure that the pressure drawdown decreases as the perforated length increases.

It can also be seen from Fig.4 that the flow regime exhibits wellbore storage and three well defined flow regions: (I) early vertical radial flow period, (2) early linear flow period, and (3) a pseudo-radial flow period. With transitions between periods. The dashed line marked AA (t_D=0.1) denote the times for the end of the initial vertical flow period before the pressure response disturbance has reached a vertical boundary. The exact time for the end of the flow regime is determined by eq. 10. During this flow period, P declines linearly with the logarithm of the elapsed time. The flow regime ends when the effect of the lower and/or upper reservoir limits is felt.

The second observed flow regime occupying the region between lines AA (t_D=10) and BB'(Td=10) is the early linear flow period when the pressure transient reaches the upper and lower boundaries of the reservoir.

The third flow regime (the pseudo-radial flow) occurs for t_D> 10.0. This flow regime is observed due to the absence of an aquifer or a gas Cap. During this flow regime, the horizontal well behaves as an enlarged vertical well. Similar to a vertical well intersected by a vertical Hydraulic fracture. The flow regimes radius is much larger than the producing length, L, of the well, so that the distance from the well's centre to the nearest external boundary must be several times L for this flow regime to occur. During this flow period, the streamlines will be horizontal in the x-y plane and directed towards the wellbore.

As shown in Fig.4, once the pseudo-radial flow starts, the horizontal well solutions for the values of L_D considered, that is the pressure drawdown are practically the same, Thus, during late times, when the well reaches pseudo- radial flow in an infinite reservoir, the performance of horizontal wells expressed in terms of productivity index, Irrespective of the well length approach the same value. Therefore, increasing the length of the horizontal drain section of a horizontal well increases the oil production initially. However, after some time, the- oil recovery decreases as the well length Increases. Hence, the incremental oil gain by increasing the well length has to be weighed against additional drilling/completion costs and the attendant operational risks.

The late-time linear flow period (the fourth flow period usually observed in horizontal wells) is not observed here because of the infinite nature of the reservoir. This regime would occur only when the pressure transient reach the lateral extremities of the reservoir and these lateral extremities are non-existent here.

One of the important applications of the method developed in this study is to evaluate the productivity of a horizontal well or of the formation. The productivity of a conventional well is proportional to the transmissibility (kh). Low productivities result from low values of k or h or both. This can be compensated in horizontal wells where the length L, is chosen by the engineer. The kL product in a horizontal well plays a similar role to that of the kh product in a vertical well If eq. 5 is evaluated at the wellbore, we obtain the productivity of the well, not that of the whole formation. Formation productivity can be obtained by evaluating eq.5 at the appropriate values of Xd ,yD, and zD,.

The use of eq.5 in the form discussed here assumes that downhole flow rate data is used(if not, wellbore storage above bottomhole pressure gauges has to be taken into account) However, in a horizontal well, there will be considerable wellbore volume below the tool even when a shut-in device is used or the down hole flow rate is measured. The storage effects due to this volume typically lasts longer than those for a vertical well in the same formation because the anisotropy reduces the effective permeability at early times to $(k_h k_x)^{1/2}$.

CONCLUSION

A numerical method which can be used to examine the effect of length on pressure drawdown in a horizontal well has been developed. The analysis method (model) was validated using an example from the literature. The results show that the pressure drawdown in a reservoir penetrated by a horizontal well decreases as the length of the horizontal drain section increases However, the relationship was found to the non-linear. At late times, when pseudo-radial flow has been achieved (in an infinite reservoir), the reduction in pressure drawdown with increase in the length of the drain hole becomes negligible. The method provides a tool that can be used to estimate the optimum length of the horizontal drain hole to achieve a balance between the cost of drilling, completion and the attendant operational risks against the well productivity. It is also useful for screening potential horizontal well candidates for detailed numerical simulation studies.

Nomenclature

q = production rate per unit length of line source barrels/day
q_w= total production, barrels
x,y,z = co-ordinate, feet
L = full well length, feet
C_t = total system compressibility, psi^{-1}
h = reservoir thickness, feet
k = permeability, millidarcy
n = k/Φμc=hydraulic diffusivity, sq.ft/hr
B = formation volume factor, res bbl/stb
_p = pressure drop, psi
p_1= initial uniform pressure in reservoir, psi
z_w = distance of the well from the boundary at z=0, feet
r_w = Wellbore radius, feet
t = time
n = number of terms in a series
t_p = dimensionless time
P_D = dimensionless pressure
P_{WD} = dimensionless wellbore pressure
y^1 = y - coordinate of the centre of well, ft
x^1 = x - coordinate of the centre of well, ft

$$G_x = \{erf \frac{(\sqrt{(}K_h / K_x) - X_D)}{2\sqrt{t_D}} + erf \frac{(\sqrt{(}K_h / K_x) + X_D)}{2\sqrt{t_D}}\}$$

$$G_y = \exp(-y_D{}^2 / 4t_D)$$

$$G_Z = \sum_{n=-\infty}^{\infty} \exp\{\frac{(-z - z_w + 2nh)^2)}{4t_D} + \exp\frac{(-z + z_{wD} + 2nh)^2}{4t_D}\}$$

$$erf(x) = (\underset{0}{2} /^{x} \sqrt{\pi}) \_ \exp(-u^2)du$$

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Appendix I

$X_D = (2(x-x^1)L)\sqrt{(K_h/k_x)})$ .... (A-1)

$y_D = (2(y-y)/L)\ \sqrt{/}(K_h/k_y)$ .... (A-2)

$Z_{WD} = z_w/h$ .... (A-3)

$R_{WD} = R_w/2L(\sqrt{(K_h/k_y)}+\sqrt{(K_h/k_y)}$ .... (A-4)

$Z_D = z_{wD} + R_{WD}$ .... (A-5)

$t_D = (0.001055K_h t)(\Phi\mu c_1 L^2)$ .... (A-6)

$P_D = (k_h h\_P)/(141.2quB)$ .... (A-7)
$L_D = (L/2h)\sqrt{(K_h/k_h)}$ .... (A-8)

Fig. 3: Flow chart For The Computer Programme

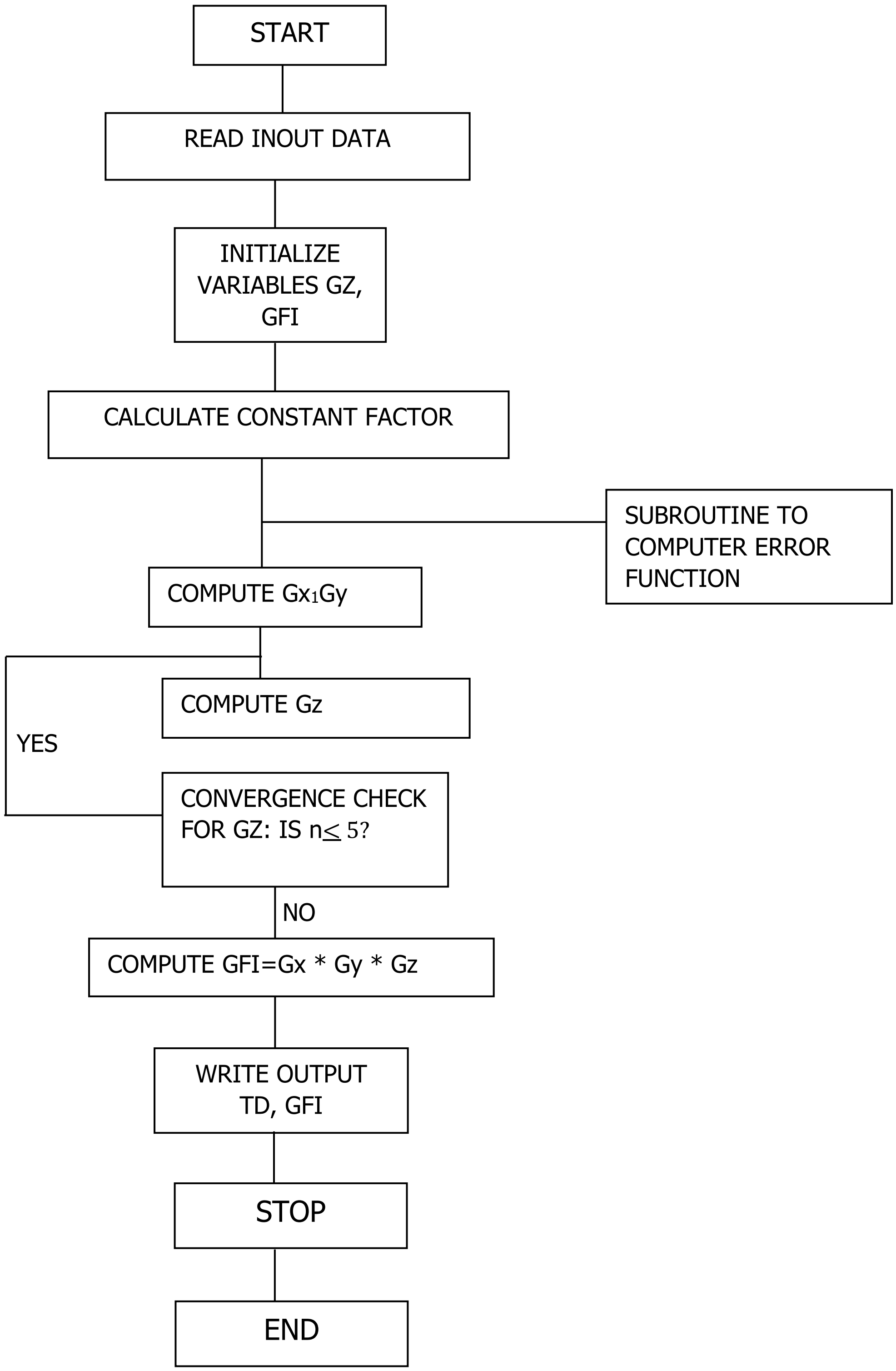

Table 1: Well And Reservoir Data

Porosity, Fraction	0.22
Oil viscosity, cp	1.0
Compressibility psi	1.0 x10^{-5}
Well length B	900
Reservoir thickness, ft	100
Production rate, stb/day	500
Wellbore radius, ft	0.3
Formation volume factor. res.bbl/stb	1,25
Horizontal Permeability, md	1.02
Initial pressure , psi	5000
Vertical permeability, md	1.05

TABLE 2
****
OUTPUT DATE
****

RESERVOIR THICKNESS = 100,00ft

PKX = 1.02md ;PKY = 1.02md; PKZ= 1.05md

LD --------------	XD --------------	YD --------------	ZD --------------	ZWD	RWD
2.5364980	0.6880000	0.0012000	0.5012000	0.5000000	0.0012000
3.5510980	0.6980000	0.0008571	0.5008571	0.5000000	0.0008571
4.5656970	0.7010000	0.0006667	0.5006667	0.5000000	0.0006667
5.8339460	0.7080000	0.0005217	0.5005217	0.5000000	0.0005217
7.6094950	0.7150000	0.0004000	0.5004000	0.5000000	0.0004000

Table 3: Pressure Drawdown Coefficient Vs. Time Horizontal Length, L(Ft)

TD *********	L1 ********* 500.0 --------------	L2 ********* 700.0 --------------	L3 ********* 900.0 --------------	L4 ********* 1150.0 --------------	L5 ********* 1500.0
.1E+00	4.78812	4.74360	4.73012	4.69840	4.66636
.2E+00	3.06414	3.03985	3.03253	3.01535	2.99806
.5E+00	1.63103	1.62296	1.62053	1.61482	1.60907
.7E+00	1.25986	1.25497	1.25349	1.25002	1.24653
.1E+01	0.94178	0.93903	0.93820	0.93624	0.93428
.2E+01	0.51276	0.51194	0.51170	0.51112	0.51053
.5E+01	0.21690	0.21676	0.21671	0.21661	0.21651
.7E+01	0.15666	0.15658	0.15656	0.15651	0.15645
.1E+02	0.11059	0.11055	0.11054	0.11051	0.11048
.2E+02	0.05584	0.05 584	0.05583	0.05583	0.05582
.5E+02	0.02247	0.02247	0.02247	0.02247	0.02247
.7E+02	0.01607	0.01607	0.01607	0.01607	0.01607
.1E+03	0.01126	0.01126	0.01126	0.01126	0.01126
.2E+03	0.00564	0.00563	0.00563	0.00563	0.00563
.5E+03	0.00226	0.00226	0.00226	0.00226	0.00226
.7E+03	0.00161	0.00161	0.00161	0.00161	0.00161
.1E+04	0.00113	0.00113	0.00113	0.00113	0.00113

Table 4: Dimensionless Wellbore Pressure Versus Dimensionless Time For A Horizontal Well In An Infinite Slab Reservoir

t_D	L_D=2.54	L_D=3.55	L_D=4.57	L_D=5.83	L_D=7.61
$1x10^{-5}$	0.2596	0.2354	0.2122	0.1879	0.1619
$1x10^{-4}$	0.4835	0.3975	0.3330	0.2865	0.2375
$1x10^{-3}$	0.7104	0.5596	0.4644	0.3 852	0.3132
$1x10^{-2}$	0.9374	0.7217	0.5904	0.4839	0.3888
$1x10^{-1}$	1.1642	0.8316	0.7165	0.5826	0.4645
$2x10^{-1}$	1.3399	1.0058	0.8902	0.75519	0.5566
$5x10^{-1}$	1.6551	1.3819	1.2026	1.0660	0.8748
$7x10^{-1}$	1.7845	1.4476	1.3312	1.1942	1.0024
$1x10^{0}$	1.9323	1.5949	1.4783	1.3410	1.1488
$2x10^{0}$	2.2578	1.9196	1.8027	1.6648	1.4717
$5x10^{0}$	2.7476	2.4087	2.2917	2.1533	2.1265
$7x10^{0}$	2.9148	2.5758	2.4588	2.3204	2.1266
$1x10^{1}$	3.0942	2.7551	2.6381	2.5000	2.3067
$2x10^{1}$	3.5936	3.2544	3.1373	2.9988	2.8049
$5x10^{1}$	3.5960	3.5568	3.4397	3.3012	3.1073
$7x10^{1}$	3.6520	3.6150	3.4981	3.3595	3.1655
$1x10^{2}$	3.6962	3.6592	3.5423	3.4037	3.2097
$2x10^{2}$	3.7489	3.7119	3.5423	3.4565	3.3364
$5x10^{2}$	3.7826	3.7456	3.5946	3.4902	3.3440
$7x10^{2}$	3.7896	3.7553	3.6287	3.4973	3.3511
$1x10^{3}$	3.7953	3.7581	3.6355	3.6118	3.5786

Table 5: Comparison Of Results

Time, t(hrs)	Dimensionless time (t_D)	Dimensionless Pressure (P_D)	_P (Ref.4), (psi)	_P Computed (psi)
1	$5.86 x 10^{-4}$	0.4064	408.6	412.6
10	$5.86 x 10^{-3}$	0.5325	520.5	521.4
100	$5.86 x 10^{-2}$	0.6585	745.7	750.9
1000	$5.86 x 10^{-1}$	1.2579	1469.5	1523.6
10000	$5.86 x 10^{0}$	2.3636	2650.9	2658.9

Table 6. Pressure Drawdown Versus Time (Q=500 Bb/D)

t(hrs)	L_D=2.54	L_D=3.55	L_D=4.57	L_D=5.83	L_D=7.61
$1.00 x 10^{-3}$	135.363	120.837	107.746	95.017	81.433
$1.00 x 10^{-2}$	243.388	217.266	193.729	170.842	146.417
$1.00 x 10^{-1}$	437.670	357.514	299.133	256.159	211.835
$1.00 x 10^{0}$	633.678	497.763	412.367	341.553	277.321
$1.00 x 10^{1}$	830.061	633.632	521.391	426.948	342.739
$1.00 x 10^{2}$	1139.751	850.858	750.894	634.208	471.333
$1.00 x 10^{3}$	19 17.268	1624.747	1523.640	1404.395	1241.426
$1.00 x 10^{4}$	3053.677	2760.209	2658.905	2539.12	2371.425
$1.00 x 10^{5}$	3237.677	3175.665	3064.784	2984.683	2872.56
$1.00 x 10^{6}$	3432.417	3370.382	3249.376	3164.463	3045.790

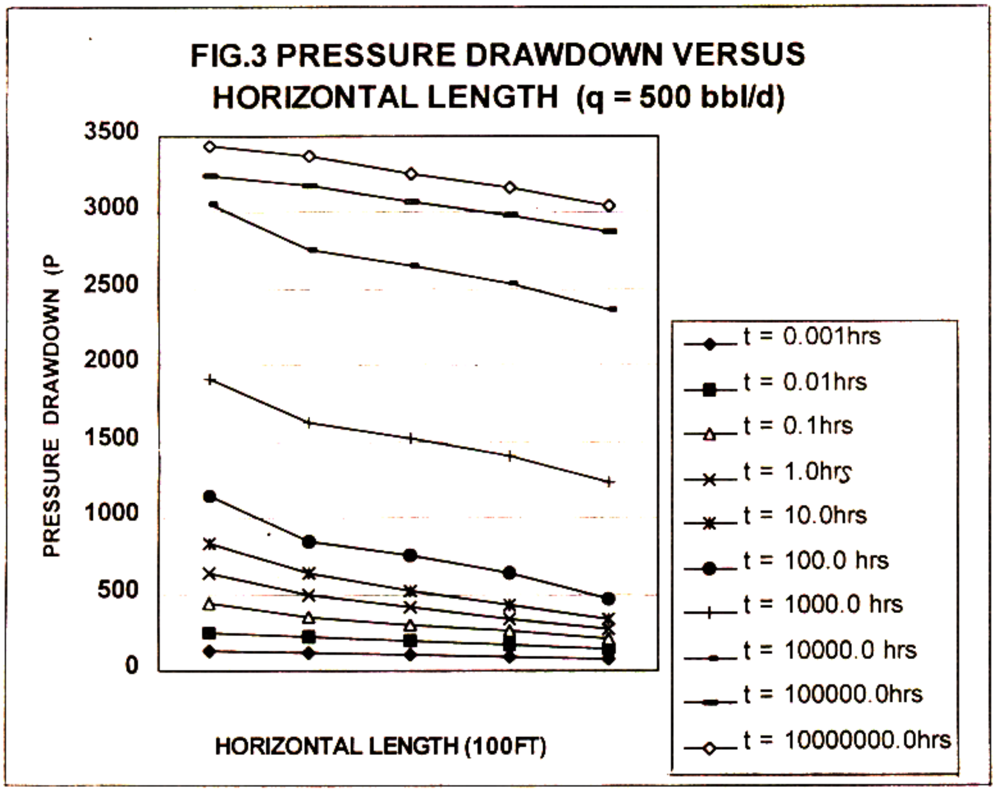
FIG.3 PRESSURE DRAWDOWN VERSUS
HORIZONTAL LENGTH (q = 500 bbl/d)
PRESSURE DRAWDOWN (P
3500
3000
2500
2000
1500
1000
500
0
HORIZONTAL LENGTH (100FT)
t = 0.001hrs
t = 0.01hrs
t = 0.1hrs
t = 1.0hrs
t = 10.0hrs
t = 100.0 hrs
t = 1000.0 hrs
t = 10000.0 hrs
t = 100000.0hrs
t = 10000000.0hrs